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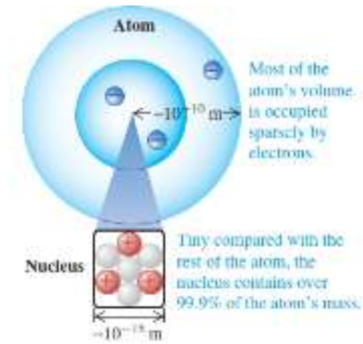
Lecture 1: Electric charge and electric field

Electric Charge and Electric Field

(Chapter 21 in textbook)



❖ *Electric Charge and the Structure of Matter*

The structure of atoms can be described in terms of three particles:

- The negatively charged **electron**

$$Mass = 9.109 \times 10^{-31} kg$$

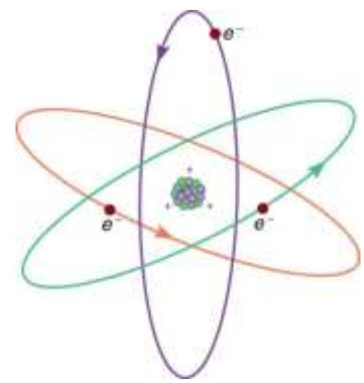
- The positively charged **proton**

$$Mass = 1.673 \times 10^{-27} kg$$

- The uncharged **neutron**

$$Mass = 1.675 \times 10^{-27} kg$$

❖ *Charge Carried by Electrons and Protons*



A model of an atom with negative electrons orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged protons.
Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.)
The charges of electrons and protons are identical in magnitude but opposite in sign. The magnitude of this basic charge is

$$q = 1.6 \times 10^{-19}\ Coulomb\ (C)$$

❖ *Conductors and insulators*

Materials that allow easy passage of charges are called conductors. (e.g. most metals)
Materials that resist electronic flow are called insulators. (e.g. glass, wood).

Coulomb's Law

The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

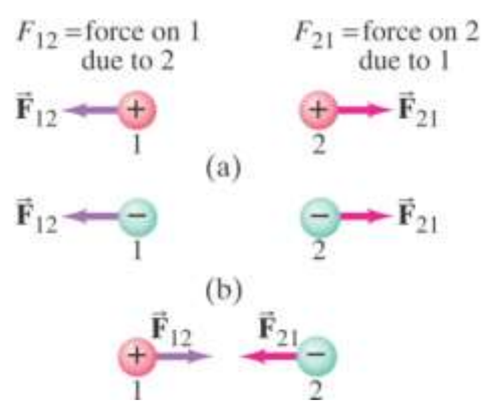
$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

Units: q_1 and q_2 are in coulombs (C); F is in newton (N).

Notes:

- The direction of F is determined using the fact that like charges repel and unlike charges attract.
- r is the distance between the two charges.
- the permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$:

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$$

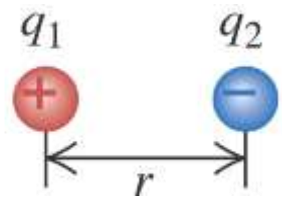


The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.

Example 1 : Forces between two point charges

Two point charges $q_1 = 25\,nC$ and $q_2 = -75\,nC$ are separated by a distance of $3.0\ cm$. Find the magnitude and direction of the electric force that q_1 exerts on q_2.

Solution:



$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$= 9\times10^9\ \frac{(25\times10^{-9})(-75\times10^{-9})}{3\times10^{-2}} = 0.0187\ N$$

The force is attractive

Example 2: Compare the strength of the electrostatic force between the electron and proton in a hydrogen atom with the corresponding gravitational force between the two. Remember that a hydrogen atom consists of a single electron in orbit around a proton. The electron is pictured as moving around the proton in a circular orbit with radius $r = 5.29 \times 10^{-11} m$.

What is the ratio of the magnitude of the electric force between the electron and proton to the magnitude of the gravitational attraction between them?

$$m_e = 9.1\times10^{-31}kg$$
$$m_p = 1.67\times10^{-27}kg$$

The gravitational constant is

$$G = 6.67\times10^{-11}Nm^2/kg^2$$

Solution: The electric force is given by Coulomb's law and the gravitational force by Newton's law of gravitation.

Each particle has charge of magnitude $e = 1.6\times10^{-19}C$.

$$\boldsymbol{F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = k\frac{e^2}{r^2}}$$
$$\boldsymbol{F_g = G\frac{m_e m_p}{r^2}}$$

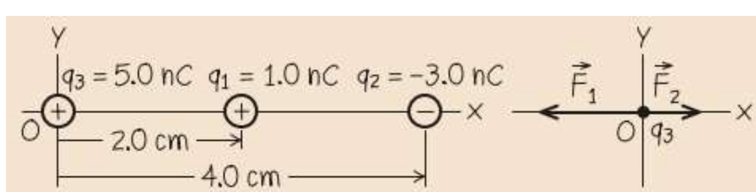
The ratio of the two forces is

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}}{G\frac{m_e m_p}{r^2}} = \frac{ke^2}{Gm_e m_p} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times9.1\times10^{-31}\times1.67\times10^{-27}}$$

$$= 2.27\times10^{39}$$

Example 3: **Vector addition (Superposition) of electric forces on a line**

Two point charges are located on the x-axis of a coordinate system: $q_1 = 1.0$ nC is at $x = +2.0$ cm, and $q_2 = -3.0$ nC is at $x = +4.0$ cm. What is the total electric force exerted by q_1 and q_2 on a charge $q_3 = 5.0$ nC at $x = 0$?



Solution:

$$F_{1\,on\,3} = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r_{13}^2}$$

$$= (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{(1.0 \times 10^{-9}\ \mathrm{C})(5.0 \times 10^{-9}\ \mathrm{C})}{(0.020\ \mathrm{m})^2}$$

$$= 1.12 \times 10^{-4}\ \mathrm{N} = 112\ \mu\mathrm{N}$$

In the same way we can show that $\boldsymbol{F_{2\,on\,3}} = 84\ \mu N$.
Thus we have:
$F_{1\,on\,3} = -112\ i \quad and\ F_{2\,on\,3} = 84\ i$
Therefore, the net force on q_3 is
$F_3 = (-112\mu N)i + (84\mu N)\ i = (-28\mu N)\ i$

Example 3: **Vector addition (Superposition) of electric forces in a plane**

Two equal positive charges $q_1 = q_2 = 2.0\ \mu\mathrm{C}$ are located at $x = 0,\ y = 0.30$ m and $x = 0,\ y = -0.30$ m, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0\ \mu\mathrm{C}$ at $x = 0.40$ m, $y = 0$?

Solution:

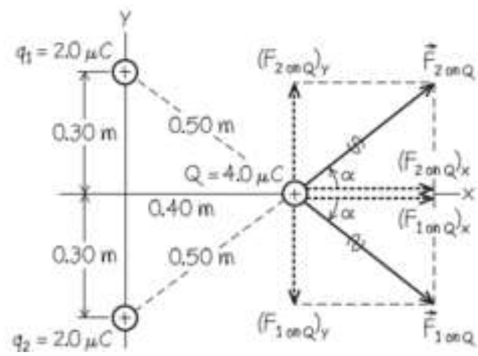
the identical charges q_1 and q_2, which are at equal distances from Q. From Coulomb's law, *both* forces have magnitude

$$F_{1\text{ or }2\text{ on }Q} = (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2}) \times \frac{(4.0 \times 10^{-6}\ \mathrm{C})(2.0 \times 10^{-6}\ \mathrm{C})}{(0.50\ \mathrm{m})^2} = 0.29\ \mathrm{N}$$

The x-components of the two forces are equal:

$$(F_{1\text{ or }2\text{ on }Q})_x = (F_{1\text{ or }2\text{ on }Q})\cos\alpha = (0.29\ \mathrm{N})\frac{0.40\ \mathrm{m}}{0.50\ \mathrm{m}} = 0.23\ \mathrm{N}$$

From symmetry we see that the y-components of the two forces are equal and opposite. Hence their sum is zero and the total force $\vec{F}$ on Q has only an x-component $F_x = 0.23\ \mathrm{N} + 0.23\ \mathrm{N} = 0.46\ \mathrm{N}$. The total force on Q is in the $+x$-direction, with magnitude 0.46 N.



❖ ***The Electric Field***

Definition of the electric field: electric force per unit charge. $\boldsymbol{E} = \frac{\boldsymbol{F}}{q_0}$ *the SI unit is N/C*

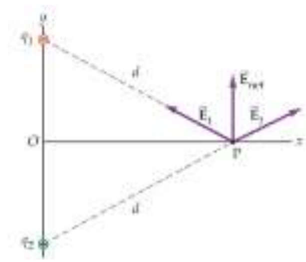
Here, q_0 is a "test charge" it serves to allow the electric force to be measured, but is not large enough to create a significant force on any other charges.

- ❖ If we know the electric field, we can calculate the force on any charge: $\boldsymbol{F = qE}$
- ❖ The direction of the force depends on the sign of the charge: in the direction of the field for a positive charge, opposite to it for a negative one.

❖ *Superposition principle for electric fields:*

Just as electric forces can be superposed, electric fields can as well.

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \cdots.$$



If we place a small test charge q_0 at the field point P, at a distance r from the source point, the magnitude of the force is given by Coulomb's law

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{r^2}$$

the magnitude of the electric field at P is

$$\boldsymbol{E} = \frac{\boldsymbol{F_0}}{q_0} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{r^2}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Example 1: What is the magnitude of the electric field at a field point 2.0 m from a point charge $q = 4.0\ nC$

Solution:

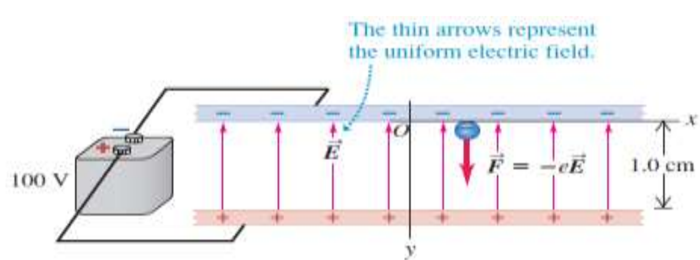
$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{4.0 \times 10^{-9}\ \mathrm{C}}{(2.0\ \mathrm{m})^2}$$
$$= 9.0\ \mathrm{N/C}$$

Example 2: When the terminals of a battery are connected to two parallel conducting plates with a small gap between them, the resulting charges on the plates produce a nearly uniform electric field between the plates. If the plates are $1\ cm$ apart and are connected to a $100\ volt$ battery. The field is vertically upward and has magnitude $E = 1 \times 10^4 N/C$.

(a) If an electron ($q = -1.6 \times 10^{-19} C\ and\ m = 9.1 \times 10^{-31} kg$) is released from rest at the upper plate, what is its acceleration?

(b) What speed and kinetic energy does it acquire while traveling 1 cm to the lower plate?

(c) How long does it take to travel this distance?



Solution: (a) Although **E** is upward (in the +y-direction), **F** is downward (because the electron's charge is negative) and so F_y is negative. Because F_y is constant, the electron's acceleration is constant:

$$\therefore\ a_y = \frac{F_y}{m} = \frac{qE}{m} = \frac{-1.6 \times 10^{-19} \times 1 \times 10^4}{9.11 \times 10^{-31}}$$
$$= -1.76 \times 10^{15} m/s^2$$

(b) The electron starts from rest, so its motion is in the y-direction only (the direction of the acceleration). We can find the electron's speed at any position y using the constant-acceleration

$$v^2 = {v_0}^2 + 2a_y(y - y_0) = 0 + 2(-1.76 \times 10^{15})(-1 \times 10^{-2} - 0)$$
$$\therefore |v| = \sqrt{0 + 2(-1.76 \times 10^{15})(-1 \times 10^{-2})} = 5.9 \times 10^6 m/s$$

The velocity is downward, so $v_y = -5.9 \times 10^6 m/s$

The electron's kinetic energy is

$$k = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31})(5.9 \times 10^6)^2 = 1.6 \times 10^{-17} J$$

(c) To calculate the time use $v_y = v_0 + a_y t$

$$t = \frac{v_y - v_0}{a_y} = \frac{-5.9 \times 10^6 - 0}{-1.76 \times 10^{15}} = 3.4 \times 10^{-9} s = 3.4\, ns$$

➢ Field of an electric dipole

Example3: Point charges q_1 and q_2 are 0.1 m apart. (Such pairs of point charges with equal magnitude and opposite sign are called electric dipoles.) Compute the electric field caused by q_1 ,the field caused by q_2 and the total field (a) at point a (b) at point b, and (c) at point c

Solution:

We must find the total electric field at various points due to two point charges. We use the principle of superposition: $\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2}$. The field points $a, b. \, and \, c$ are shown in the figure.

EXECUTE: At each field point, E depends on $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$there; we first calculate the magnitudes $\boldsymbol{E_1}$and $\boldsymbol{E_2}$ at each field point. At a the magnitude of the field E_{1a} caused by q_1 is

$$E_{1a} = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9} \text{ C}}{(0.060 \text{ m})^2}$$

$$= 3.0 \times 10^4 \text{ N/C}$$

We calculate the other field magnitudes in a similar way. The results are

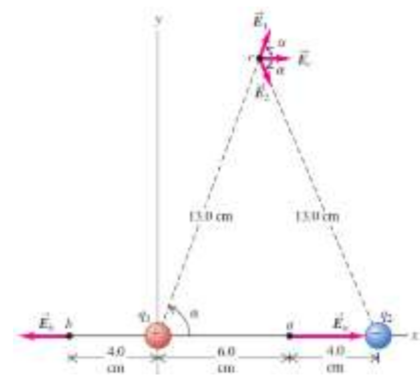
$E_{1a} = 3.0 \times 10^4$ N/C $\quad E_{1b} = 6.8 \times 10^4$ N/C

$E_{1c} = 6.39 \times 10^3$ N/C

$E_{2a} = 6.8 \times 10^4$ N/C $\quad E_{2b} = 0.55 \times 10^4$ N/C

$E_{2c} = E_{1c} = 6.39 \times 10^3$ N/C

The *directions* of the corresponding fields are in all cases *away* from the positive charge q_1 and *toward* the negative charge q_2.



the directions of $\boldsymbol{E_1}$and $\boldsymbol{E_2}$ at c. Both vectors have the same x-component:

$$E_{1cx} = E_{2cx} = E_{1c}\cos\alpha = (6.39 \times 10^3 \text{ N/C})\left(\tfrac{5}{13}\right)$$

$$= 2.46 \times 10^3 \text{ N/C}$$

From symmetry, E_{1y} and E_{2y} are equal and opposite, so their sum is zero. Hence

$$\vec{E}_c = 2(2.46 \times 10^3 \text{ N/C})\hat{\imath} = (4.9 \times 10^3 \text{ N/C})\hat{\imath}$$

➢ Field of a ring of charge

Example 4: Charge is uniformly distributed around a conducting ring of radius. Find the electric field at a point *P* on the ring axis at a distance *x* from its center.

Solution: To calculate E_x, divide the ring into small segments ds, so the electric field at P due to the segment ds is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}$$

The x-component of this field is

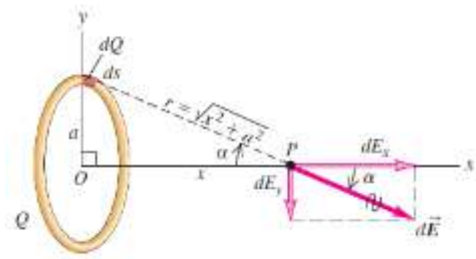
$$dE_x = dE\cos\alpha.$$

The charge on the segment ds is

$$dQ = \lambda\, ds,$$

where λ is the linear charge density

$$\lambda = Q/2\pi a$$



$$r^2 = x^2 + a^2$$

$$\cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2+a^2}}$$

$$\therefore dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{x\lambda\, ds}{(x^2+a^2)^{3/2}}$$

To find E_x we integrate this expression over the entire ring circumference that is, for s from 0 to $2\pi a$.

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(x^2+a^2)^{3/2}}\int_0^{2\pi a} ds$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(x^2+a^2)^{\frac{3}{2}}}(2\pi a) = \frac{1}{4\pi\varepsilon_0}\frac{x(\frac{Q}{2\pi a})}{(x^2+a^2)^{3/2}}(2\pi a)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+a^2)^{\frac{3}{2}}}$$ in the $+x-direction$.

➢ Field of a uniformly charged disk

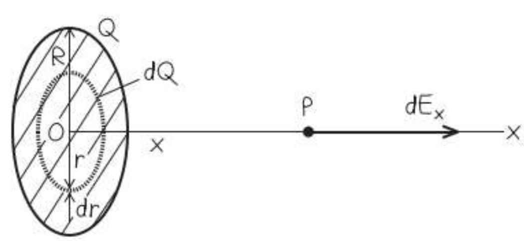
Example 5: A non-conducting disk of radius R has a uniform positive surface charge density σ . Find the electric field at a point along the axis of the disk a distance x from its center. Assume that x is positive.

Solution: the disk is a set of concentric rings. A typical ring has a charge , inner radius r, and outer radius $r + dr$.

$$dA = 2\pi r dr$$

The charge per unit surface area is $\sigma = \frac{dQ}{dA}$, so the charge of the ring is

$$dQ = \sigma\, dA = 2\pi\sigma r dr$$



The field component dE_x at point P due to this ring (Similar to example 4 and replacing the ring radius a with r.) is

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{x\sigma\, dA}{(x^2+r^2)^{3/2}}$$

To find the total field due to all the rings, we integrate dE_x over r, from $r = 0\ to\ r = R$

$$E_x = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{x(2\pi\sigma r dr)}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{4\varepsilon_0}\int_0^R \frac{2rdr}{(x^2+r^2)^{3/2}}$$

Let $t = x^2 + r^2$, so $dt = 2rdr$, the result is

$$E_x = \frac{\sigma x}{4\varepsilon_0}\left[-\frac{1}{\sqrt{x^2+R^2}} + \frac{1}{x}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2)+1}}\right]$$

Note that if the disk is very large (or we are very close to it), so that $R \gg x$, the term $\frac{1}{\sqrt{(R^2/x^2)+1}}$ will be much less than 1. then the field becomes

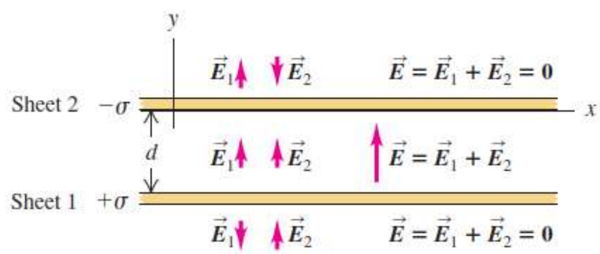
$$E = \frac{\sigma}{2\varepsilon_0}$$

This result shows that for an infinite plane sheet of charge the field is independent of the distance from the sheet.

The direction of the field is perpendicularly away from the sheet.

- **Field of two oppositely charged infinite sheets**

Example 6: Two infinite plane sheets with uniform surface charge densities and are placed parallel to each other with separation. Find the electric field between the sheets, above the upper sheet, and below the lower sheet.



Solution: both $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ have the same magnitude at all points, independent of distance from either sheet.

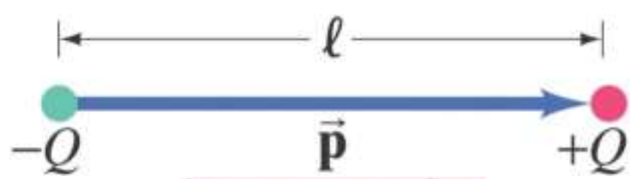
$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

E_1 is everywhere directed away from sheet 1(+ charge), and $E_2$is everywhere directed toward sheet 2 (-charge).

Between the sheets, $E_1$and E_2 reinforce each other; above the upper sheet and below the lower sheet, they cancel each other. Thus the total field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \frac{\sigma}{\epsilon_0}\hat{j} & \text{between the sheets} \\ 0 & \text{below the lower sheet} \end{cases}$$

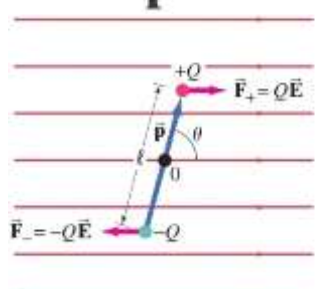
- **Electric Dipoles**



An electric dipole consists of two charges Q, equal in magnitude and opposite in sign, separated by a distance l .

The dipole moment, $\mathbf{p} = Ql$, points from the negative to the positive charge.

An electric dipole in a uniform electric field will experience no net force, but it will, in general, experience a torque:



$$\tau = QE\frac{\ell}{2}\sin\theta + QE\frac{\ell}{2}\sin\theta = pE\sin\theta.$$

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}.$$

- The torque is maximum when and $\boldsymbol{p}$ and $\boldsymbol{E}$ are perpendicular and is zero when they are parallel or antiparallel.
  - The torque always tends to turn $\boldsymbol{p}$ to line up with $\boldsymbol{E}$.

- The position of stable equilibrium occurs when $\varphi = 0$ (p and E are parallel) and when $\varphi = \pi$ (p and E are antiparallel) is a position of unstable equilibrium.

❖ Potential Energy of an Electric Dipole

When a dipole changes direction in an electric field, the electric-field torque does work on it, with a corresponding change in potential energy.

The work done by a torque during an infinitesimal displacement is $d\theta$ is given by

$$dW = \tau\, d\theta = -pEsin\theta$$

In a finite displacement from θ_1 to θ_2 the total work done on the dipole is

$$W = \int_{\theta_1}^{\theta_2} (-pEsin\theta)\, d\theta$$

$$W = pEcos\theta_2 - \mathrm{pEcos}\theta_1$$

The work is the negative of the change of potential energy

$$W = U_1 - U_2$$

So a suitable definition of potential energy for this system is

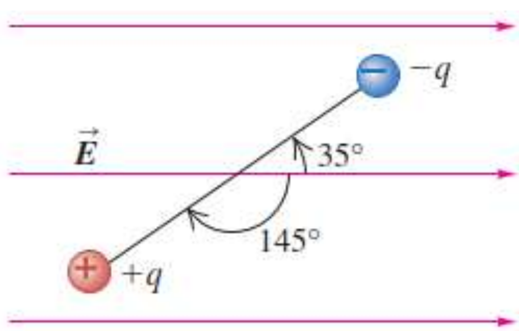
$$U(\theta) = -pEcos\theta$$

Since $pEcos\theta = p \cdot E$ (scalar product)

$$\therefore \quad U = -p \cdot E$$

- The potential energy has its minimum (most negative) value $U = -pE$ at the stable equilibrium position, where $\theta = 0$ and $\boldsymbol{p}$ is parallel to $\boldsymbol{E}$
- The potential energy is maximum when $\theta = \pi$ and **p** is antiparallel to **E** then $U = +pE$
- A $\theta = \pi/2$ t where $\boldsymbol{p}$ is perpendicular to $\boldsymbol{E}$, $\boldsymbol{U = 0}$

Example: the figure shows an electric dipole in a uniform electric field of magnitude $5 \times 10^5\ N/C$ that is directed parallel to the plane of the figure. The charges are $\mp 1.6 \times 10^{-19} C$; both lie in the plane and are separated by $0.125\ mm$.



Find:

(a) The net force exerted by the field on the dipole.

(b) The magnitude and direction of the dipole moment.

(c) The magnitude and direction of the torque.

(d) The potential energy of the system in the position shown.

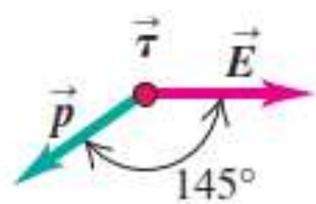
<u>Solution:</u>

(a) The field is uniform, so the forces on the two charges are equal and opposite. Hence the total force on the dipole is zero.

(b) the magnitude of the electric dipole moment is

$p = qd = 1.6 \times 10^{-19} \times 0.125 \times 10^{-3} = 2 \times 10^{-29} C.m$

The direction of $\boldsymbol{p}$ is from negative to positive charge, 145^0 clockwise from the direction of the electric field.



(c) The magnitude of the torque is

$$\tau = pEsin\theta = 2 \times 10^{-29} \times 5 \times 10^5 \times \sin 145^0$$

$$= 5.7 \times 10^{-24} N.m$$

The direction of the torque $\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E}$ is out of the page (right hand rule).this corresponds to a counterclockwise torque that tends to align $\boldsymbol{p}$ with $\boldsymbol{E}$.

(d) The potential energy is

$U = -pE\cos\theta = -2 \times 10^{-29} \times 5 \times 10^{5} \times \cos 145^{0} \ = 8.2 \times 10^{-24}\,J$

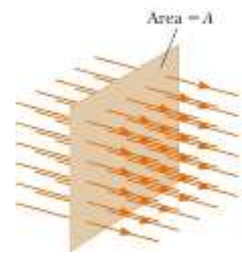
Lecture 2: Gauss's Law

GAUSS'S LAW

Gauss's law is an alternative to *Coulomb's law*.
It provides a different way to express the relationship between electric charge and electric field.

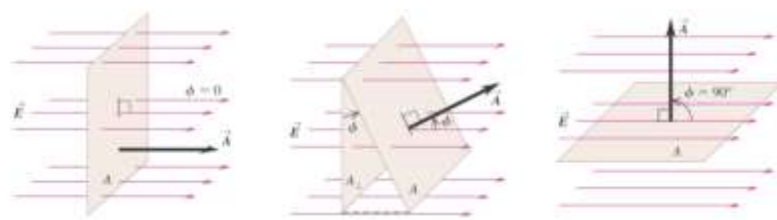
Definition: The total electric flux through any closed surface is proportional to the total electric charge inside the surface.

❖ **Calculating the electric flux**



The ***Electric Flux*** (Φ_E) is defined as the product of the magnitude of the electric field **E** and the surface area, **A**, perpendicular to the field.

- For a uniform electric field: $\Phi_E = EA$
- **Flux Units:** $N \cdot m^2/C$



$\Phi_E = E{\cdot}A$ $\Phi_E = E{\cdot}A{\cdot}\cos\varphi$ $\Phi_E = 0$

- Since the **electric flux** Φ_E through a cross sectional area **A** is proportional to the total number of field lines crossing the area.
- If the area is flat but not perpendicular to the field then fewer field lines pass through it, then

$$\Phi_E = EA\cos\varphi = E \cdot A = E_{\perp}A$$

- Φ_E is a maximum when the surface is perpendicular to the field: **θ = 0°**
- Φ_E is zero when the surface is parallel to the field: **θ = 90°**
- If the field varies over the surface, $\Phi_E = EA\cos\theta$ is valid for only a small element of the area.

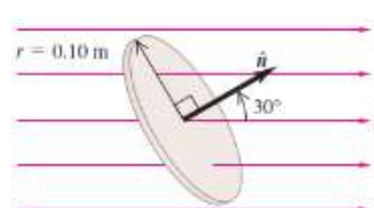
- For a Non-uniform Electric Field:

What happens if the electric field E isn't uniform but varies from point to point over the area A? Or what if A is part of a curved surface? Then we divide A into many small elements dA. Then we get the general definition of flux.

$$\Phi_E = \int E\cos\phi \, dA = \int E_{\perp}\, dA = \int \vec{E} \cdot d\vec{A} \quad \text{(general definition of electric flux)}$$

We call this integral the ***surface integral*** of the component $E_{\perp}$ over the area, or the ***surface integral*** of $\boldsymbol{E \cdot dA}$.

Example 1: A disk of radius 0.10 m is oriented with its normal unit vector n at 30^{0} to a uniform electric field of magnitude $2 \times 10^{3} n/C$. (Since this isn't



a closed surface, it has no “inside” or “outside.” That’s why we have to specify the direction of in the figure.)

(a) What is the electric flux through the disk?

(b) What is the flux through the disk if it is turned so that $\boldsymbol{n}$ is perpendicular to $\boldsymbol{E}$?

(c) What is the flux through the disk if $\boldsymbol{n}$ is parallel to $\boldsymbol{E}$?

Solution:

(a) The area $A = \pi(0.1)^2 = 0.0314\ m^2$

$$\Phi_E = EA\cos\theta$$

$$= 2 \times 10^3 \times 0.0314\cos(30^0) = 54\ N.m^2/C$$

(b) The normal to the disk is now perpendicular to E, so $\varphi = 90^0,\ and\ \cos 90^0 = 0$

$$\therefore E = 0$$

(c) The normal to the disk is parallel to E, so $\varphi = 0,\ and\ \cos\varphi = 1.$

$$\therefore \Phi_E = 2 \times 10^3 \times 0.0314 \times 1 = 63\ N.\frac{m^2}{C}$$

- **Electric flux through a cube**

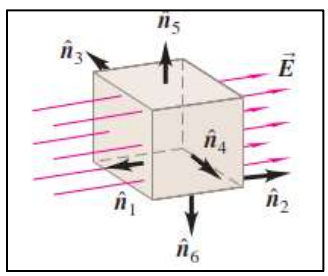
Example 2: An imaginary cubical surface of side L is in a region of uniform electric field $\boldsymbol{E}$. Find the electric flux through each face of the cube and the total flux through the cube when

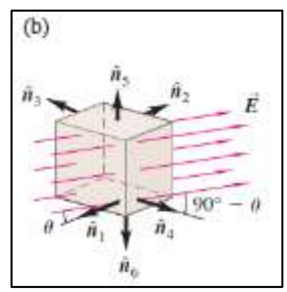
(a) it is oriented with two of its faces perpendicular to $\boldsymbol{E}$.

(b) the cube is turned by an angle θ about a vertical axis.

Solution:

(a) The angle between $\boldsymbol{n_1}$ and $\boldsymbol{E}$ is 180°, the angle between $\boldsymbol{E}$ and $\boldsymbol{n_2}$ is 0^0, and the angle between $\boldsymbol{E}$ and each of the other four unit vectors is 90°. Each face of the cube has area L^2 so the fluxes through the faces are:



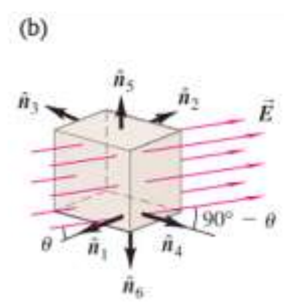


$$\varphi_{E1} = E \cdot A_1 = EL^2\cos 180^0 = -\ EL^2$$
$$\varphi_{E2} = E \cdot A_2 = EL^2\cos 0^0 = +\ EL^2$$
$$\varphi_{E3} = \varphi_{E4} = \varphi_{E5} = \varphi_{E6} = EL^2\cos 90^0 = 0$$

The total flux through the cube is

$$\varphi_E = \varphi_{E1} + \varphi_{E2} + \varphi_{E3} + \varphi_{E4} + \varphi_{E5} + \varphi_{E6}$$
$$= -EL^2 + EL^2 + 0 + 0 + 0 + 0 = 0$$
$$\varphi_{E1} = E \cdot A_1 = EL^2\cos(180^0 - \theta) = -\ EL^2\cos\theta$$
$$\varphi_{E2} = E \cdot A_2 = EL^2\cos\theta = +\ EL^2\cos\theta$$
$$\varphi_{E3} = E \cdot A_3 = EL^2\cos(90^0 + \theta) = -\ EL^2\sin\theta$$
$$\varphi_{E4} = E \cdot A_4 = EL^2\cos(90^0 - \theta) = +\ EL^2\sin\theta$$

(b) The field $\boldsymbol{E}$ is directed into faces 1 and 3, so the fluxes through them are negative; $\boldsymbol{E}$ is directed out of faces 2 and 4, so the fluxes through them are positive. We find



$$\varphi_{E5} = E \cdot A_5 = EL^2\cos 90^0 = 0$$

The total flux

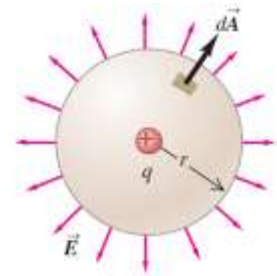
$$\varphi_E = \varphi_{E1} + \varphi_{E2} + \varphi_{E3} + \varphi_{E4} + \varphi_{E5} + \varphi_{E6} = 0,$$

through the surface of the cube is again zero

- **Electric flux through a sphere**

Example: A point charge $q = +3\ \mu C$ is surrounded by an imaginary sphere of radius $r = 0.2\ m$ centered on the charge. Find the resulting electric flux through the sphere.

Solution: The electric flux $\Phi_E = \int E dA$, but the magnitude of the electric field $\boldsymbol{E}$ is the same at every point on the surface of the sphere.



$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\therefore \Phi_E = E\int dA = EA = \frac{q}{4\pi\epsilon_0 r^2} \times 4\pi r^2 = \frac{q}{\epsilon_0} = \frac{3\times10^{-6}}{8.85\times10^{-12}}$$
$$= 3.4\times10^{5} N.m^2/C$$

The flux through any surface enclosing a single point charge is independent of the shape or size of the surface

❖ **Point Charge Inside a Spherical Surface:**

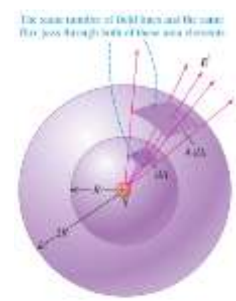
Place a single positive point charge at the center of an imaginary spherical surface with radius R. The magnitude E of the electric field at every point on the surface is given by

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$

At each point on the surface, $\boldsymbol{E}$ is perpendicular to the surface, and its magnitude is the same at every point. The total electric flux is

$$\Phi_E = EA = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}(4\pi R^2) = \frac{q}{\epsilon_0}$$

The flux is independent of the radius R of the sphere. It depends only on the charge q enclosed by the sphere.



In terms of field lines the Figure shows two spheres with radii R and $2R$ centered on the point charge. Every field line that passes through the smaller sphere also passes through the larger sphere, so the total flux through each sphere is the same.

$R \rightarrow projected\ area\ is\ dA$

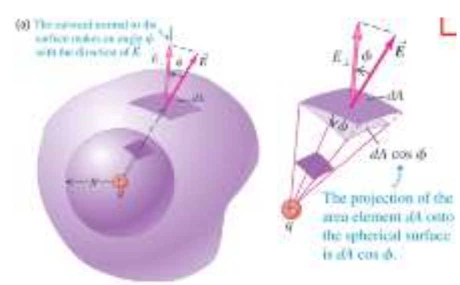
$2R \rightarrow projected\ area\ is\ 4dA$

Since $E = \left(\frac{q}{4\pi\epsilon_0}\right)\ \frac{1}{R^2}$

Hence the electric flux is the same for both areas and is independent of the radius of the sphere.

❖ **Point Charge Inside a Nonspherical Surface:**

- ➢ Divide irregular surface into dA elements, compute electric flux for each ($E\ dA\ \cos\varphi$) and sum results by integrating.
- ➢ Each dA projects onto a spherical surface element gives total electric flux through irregular surface = flux through sphere.



$\Phi_E = \oint \boldsymbol{E}\cdot \boldsymbol{dA} = \frac{q}{\epsilon_0}$, the circle means that the integral is through a closed surface. This is valid for positive or negative charge.

➢ ***If enclosed*** $\boldsymbol{q = 0}$ ***, then*** $\boldsymbol{\Phi_E = 0}$

- **General form of Gauss's law**

Suppose the surface encloses several charges. Let Q_{encl} be the total charge enclosed by the surface

$$Q_{encl} = q_1 + q_2 + q_3 + \cdots$$

Also let $\boldsymbol{E}$ be the total field at the position of the surface area element dA

$$E = E_1 + E_2 + E_3 + \cdots$$

and let $E_\perp$ be its component perpendicular to the plane of that element dA.

Then we can calculate the flux for each charge and its corresponding field and add the results. When we do, we obtain the general statement of Gauss's law:

$$\Phi_E = \oint \boldsymbol{E} \cdot \boldsymbol{dA} = \frac{Q_{encl}}{\epsilon_0}$$

Gauss's law: ***The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by*** ϵ_0

We often refer to a closed surface used in Gauss's law as a *Gaussian surface*. We can express Gauss's law in the following equivalent forms:

$$\Phi_E = \oint E\cos\phi\, dA = \oint E_\perp\, dA = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \quad \text{(various forms of Gauss's law)}$$

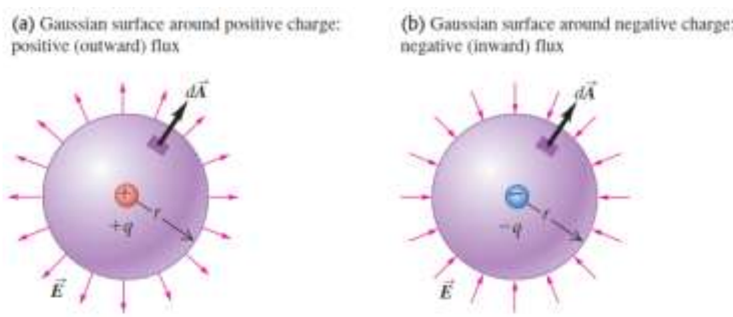
The various forms of the integral all express the same thing, the total electric flux through the Gaussian surface, in different terms.

As an example, a spherical Gaussian surface of radius r around a positive point charge $+q$ The electric field points out of the Gaussian surface, so at every point on the surface $\boldsymbol{E}$ is in the same direction as dA, $\varphi = 0$, and $E_\perp$ is equal to the field magnitude $E = q/4\pi\epsilon_0$. Since E is the same at all points on the surface. Then

$$\Phi_E = \oint E_\perp \boldsymbol{dA} = \oint \left(\frac{q}{4\pi\epsilon_0 r^2}\right) dA = \left(\frac{q}{4\pi\epsilon_0 r^2}\right) \oint dA = \left(\frac{q}{4\pi\epsilon_0 r^2}\right) A$$

$$= \left(\frac{q}{4\pi\epsilon_0 r^2}\right) 4\pi r^2 = \frac{q}{\epsilon_0}$$

The enclosed charge Q_{encl} is just the charge $+q$ so this agrees with Gauss's law.



If the Gaussian surface encloses a *negative* point charge, then $\boldsymbol{E}$ points *into* the surface at each point in the direction opposite dA, $\varphi = 180^0$ and $E_\perp$ is equal to the negative of the field magnitude $E_\perp = -E = -\left(\frac{|-q|}{4\pi\epsilon_0 r^2}\right) = -q/4\pi\epsilon_0 r^2$

$$\Phi_E = \oint E_\perp \boldsymbol{dA} = \oint \left(\frac{-q}{4\pi\epsilon_0 r^2}\right) dA = \left(\frac{-q}{4\pi\epsilon_0 r^2}\right) \oint dA = \left(\frac{-q}{4\pi\epsilon_0 r^2}\right) A$$

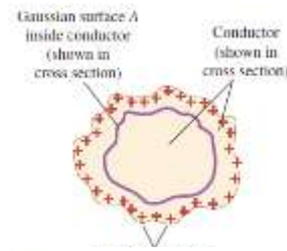
$$= \left(\frac{-q}{4\pi\epsilon_0 r^2}\right) 4\pi r^2 = \frac{-q}{\epsilon_0}$$

Lecture 3: Applications of Gauss's Law

Applications of Gauss's Law

- Gauss's law is valid for *any* distribution of charges and for *any* closed surface.
- Gauss's law can be used in two ways.
  - If we know the charge distribution, and if it has enough symmetry to let us evaluate the integral in Gauss's law, we can find the field. Or

- if we know the field, we can use Gauss's law to find the charge distribution, such as charges on conducting surfaces.

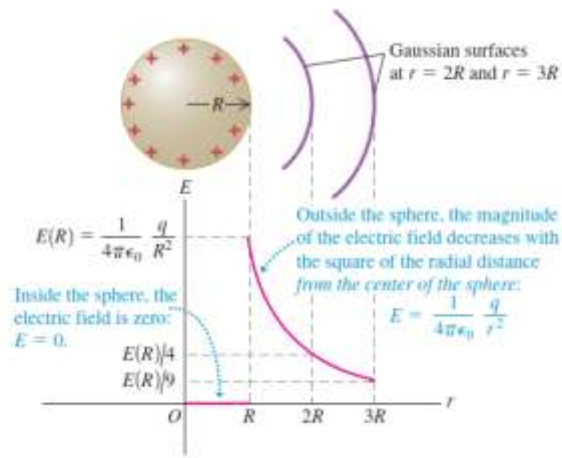


- When excess charge is placed on a solid conductor and is at rest, it resides entirely on the surface, not in the interior of the material.

(By excess we mean charges other than the ions and free electrons that make up the neutral conductor.)

❖ **Field of a charged conducting sphere**

Example: We place a total positive charge q on a solid conducting sphere with radius R . Find the electric field at any point inside or outside the sphere.



All the charge must be on the surface of the sphere. The charge is free to move on the conductor, and there is no preferred position on the surface; the charge is therefore distributed uniformly over the surface, and the system is spherically symmetric.

To exploit this symmetry, we take as our Gaussian surface a sphere of radius r centered on the conductor.

- For $r > R$ the entire conductor is within the Gaussian surface, so the enclosed charge is q, and $\boldsymbol{E}$ is uniform over the surface and perpendicular to it at each point.
  $Q_{encl} = q, \; A = 4\pi r^2, E_{\perp} = E$, the electric flux is given by:

$$\Phi_E = \oint E_{\perp} \cdot \boldsymbol{dA} = EA = \frac{Q_{encl}}{\epsilon_0} \qquad \therefore EA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

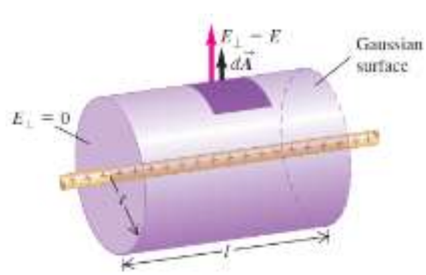
Thus, the field outside the conductor is: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

- At the surface of the sphere, where $r = R$: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$

- For $r < R$ we again have $(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0}$, But now our Gaussian surface (which lies entirely within the conductor) encloses no charge $Q_{encl} = 0$.
  The electric field inside the conductor is therefore zero.

❖ **Field of a uniform line charge**

Example 1: Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is (assumed positive). Find the electric field using Gauss's law



Solution: The flux through the flat ends of our Gaussian surface is zero because the radial electric field is parallel to these ends

On the cylindrical part of our surface we have $E_{\perp} = E \; (everywhere)$.

$Q_{encl} = \lambda l, \;\; A = 2\pi r l$

$$\Phi_E = \oint E_\perp \cdot dA = EA = \frac{Q_{encl}}{\epsilon_0}$$

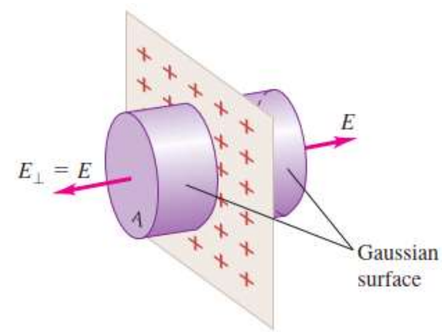
$$\therefore 2\pi r l E = \frac{\lambda l}{\epsilon_0}$$

Electric field of an infinite line of charge is $E = \frac{\lambda}{2\pi\epsilon_0 r}$

- **Field of an infinite plane sheet of charge**

Example 2: Use Gauss's law to find the electric field caused by a thin, flat, infinite sheet with a uniform positive surface charge density σ.

Solution: The flux through the cylindrical part of our Gaussian surface is zero because E is parallel to the surface. The flux through each flat end of the surface is $+EA$. The total enclosed charge is $Q_{encl} = \sigma A$
and so from Gauss's law,



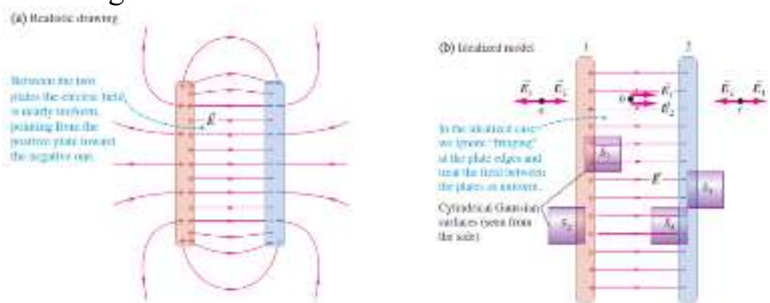
$$EA \times 2 = \frac{\sigma A}{\epsilon_0}$$

Therefore, the field of an infinite sheet of charge

$$E = \frac{\sigma}{2\epsilon_0}$$

Field between oppositely charged parallel conducting plates

Example 3: Two large plane parallel conducting plates are given charges of equal magnitude and opposite sign; the surface charge densities are $+\sigma$ and $-\sigma$ Find the electric field in the region between the plates.



The left-hand end of surface S_1 is within the positive plate 1. Since the field is zero within the volume of any solid conductor under electrostatic conditions, there is no electric flux through this end. The electric field between the plates is perpendicular to the right-hand end, so on that end, $E_\perp$ is equal to E and the flux is EA; this is positive, since $\boldsymbol{E}$ is directed out of the Gaussian surface. There is no flux through the side walls of the

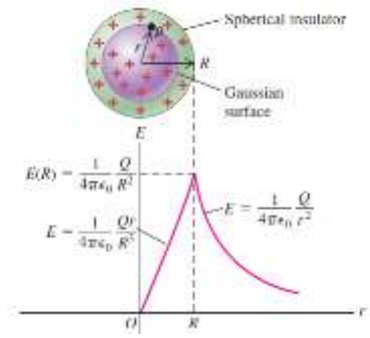
cylinder, since these walls are parallel to $\boldsymbol{E}$. So the total flux integral in Gauss's law is EA. The net charge enclosed by the cylinder is σA, so $EA = \sigma A/\epsilon_0$. Thus, the field between oppositely charged conducting plates is

$$E = \frac{\sigma}{\epsilon_0}$$

The field is uniform and perpendicular to the plates, and its magnitude is independent of the distance from either plate. The Gaussian surface S_4 yields the same result. Surfaces S_2 and S_3 yield $E = 0$ to the left of plate 1 and to the right of plate 2, respectively.

- **Field of a uniformly charged sphere**

Example: Positive electric charge is distributed uniformly *throughout the volume* of an *insulating* sphere with radius Find the magnitude of the electric field at a point a distance from the center of the sphere.



Solution: From symmetry, the direction of E is radial at every point on the Gaussian surface, so $E_{\perp} = E$ and the field magnitude is the same at every point on the surface. Hence the total electric flux through the Gaussian surface is the product of E and the total area of the surface $A = 4\pi r^2$, that is $\Phi_E = 4\pi r^2 E$

The amount of charge enclosed within the Gaussian surface depends on r. To find E inside the sphere, we choose $r < R$. The volume charge density ρ is the charge Q divided by the volume of the entire charged sphere of radius R .

$$\rho = \frac{Q}{4\pi R^3/3}$$

The volume V_{encl} enclosed by the Gaussian surface is $\frac{4}{3}\pi r^3$, so the total charge Q_{encl} enclosed by the surface is

$$Q_{encl} = \rho V_{encl} = \left(\frac{Q}{4\pi R^3/3}\right)\frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3}$$

The Gaussian law becomes

$$\frac{4}{3}\pi r^2 E = \frac{Q}{\epsilon_0}\frac{r^3}{R^3}$$

Or the field inside a uniformly charge sphere

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$$

The field magnitude is proportional to the distance r of the field point from the center of the sphere .

To find E outside the sphere, $r > R$. This surface encloses the entire charged sphere, so $Q_{encl} = Q$ and Gauss's law gives

$$4\pi r^2 E = \frac{Q}{\epsilon_0}$$

The field outside a uniformly charged sphere is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

- **Charge on a hollow sphere**

Example : A thin-walled, hollow sphere of radius 0.250 m has an unknown charge distributed uniformly over its surface. At a distance of 0.3 m from the center of the sphere, the electric field points radially inward and has magnitude $1.8 \times 10^2 N/C$. How much charge is on the sphere?

Solution: The charge distribution is the same as if the charge were on the surface of a $0.25m$ radius conducting sphere. Also, the electric field here is directed toward the sphere, so that q must be *negative.* Furthermore, the electric field is directed into the Gaussian surface, so that $E_{\perp} = -E$ and

$$\Phi_E = \oint E_{\perp} \cdot dA = E(2\pi r^2)$$

By Gauss's law, the flux is $\Phi_E = \frac{q}{\epsilon_0} = -E(2\pi r^2)$

$$\therefore q = -E \times 2\pi\epsilon_0 r^2$$
$$= -1.8 \times 10^2 \times 2\pi \times 8.85 \times 10^{-12} \times (0.3)^2$$
$$q = -1.8 \times 10^{-9} C$$

- **Electric field of the earth**

Example: The earth (a conductor) has a net electric charge. The resulting electric field near the surface has an average value of about 150 N/C directed toward the center of the earth. $R_E = 6.38 \times 10^6 m$.

(a) What is the corresponding surface charge density?
(b) What is the *total* surface charge of the earth?

Solution: (a) since $\boldsymbol{E}$ is directed into the surface, then σ is negative, and so $E_\perp = -E$.

$$\therefore \sigma = \epsilon_0 E_\perp = 8.85 \times 10^{-12} \times (-150)$$
$$= -1.33 \times 10^{-9} C/m^2$$

(b) σ is the charge per unit surface area.

$\therefore$ *the total surface chrage* $Q = 4\pi {R_E}^2 \sigma$ (or $Q = 4\pi\epsilon_0 {R_E}^2 E_\perp$)

$$Q = 4\pi \times (6.38 \times 10^6)^2 \times (-1.33 \times 10^{-9})$$
$$= -6.8 \times 10^5\, C$$

Lecture 4: Electric Potential

Electric Potential

Review:

1. Work done by a force to move a particle from point a to point b is

$$W_{a\to b} = \int_a^b F.dl = \int_a^b F \cos\theta\, dl$$

2. The work-energy theorem**:** $W_{tot} = \Delta K = K_b - K_a$

- If the force is conservative, then

$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U = mgh$$

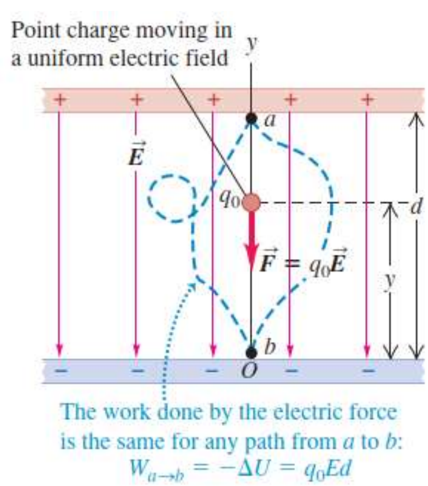
3. Conservation of energy**:** $K_a + U_a = K_b + U_b$

Electric Potential Energy

- When a charged particle moves in an electric field, the field exerts a force that can do work on the particle. The work can be expressed in terms of electric potential energy.
- Electric potential energy depends only on the position of the charged particle in the electric field.

Electric Potential Energy in a Uniform Field:

A pair of charged parallel metal plates sets up a uniform, downward electric field with magnitude $\boldsymbol{E}$. The field exerts a downward force with magnitude $F = q_0 E$ on a positive test charge q_0 . As the charge moves downward a distance d from point a to point b, the force on the test charge is constant and independent of its location. So the work done by the electric field is the product of the force magnitude and the component of displacement in the (downward) direction of the force:



$$W_{a\to b} = F\,d = q_0 E\,d$$

This work is positive, since the force is in the same direction as the net displacement of the test charge.

The force exerted on by the uniform electric field is conservative, just as is the gravitational force. This means that the work done by the field is independent of the path the particle takes from a to b. We can represent this work with a potential-energy function U, just as we did for gravitational potential energy. The potential energy for the gravitational force was $(F_y = -mg)$ was $U = mgy$, hence the potential energy for the electric force $(F_y = -q_0E)$ is

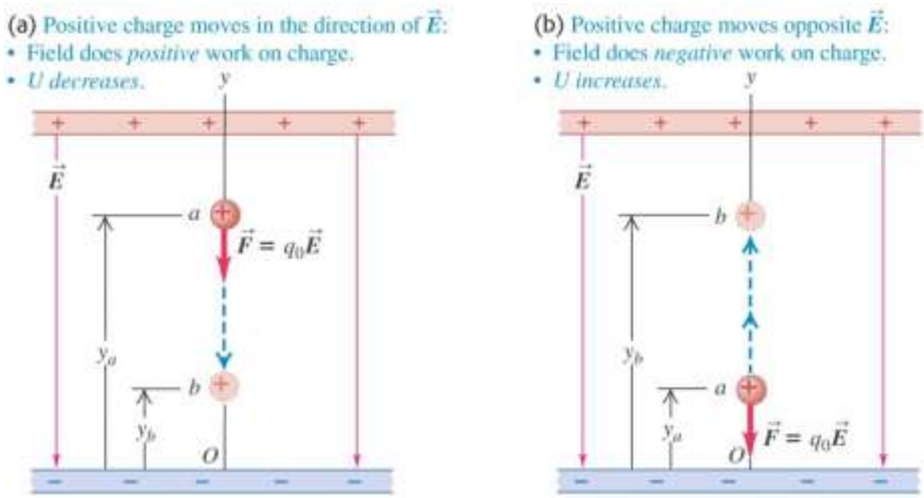
$$U = q_{0E}\, y$$

When the test charge q_0 moves from height y_a to height y_b the work done on the charge by the field is given by

$$W_{a\to b} = -\Delta U = -(U_b - U_a) = -(q_0Ey_b - q_0Ey_a) = q_0E(y_a - y_b)$$

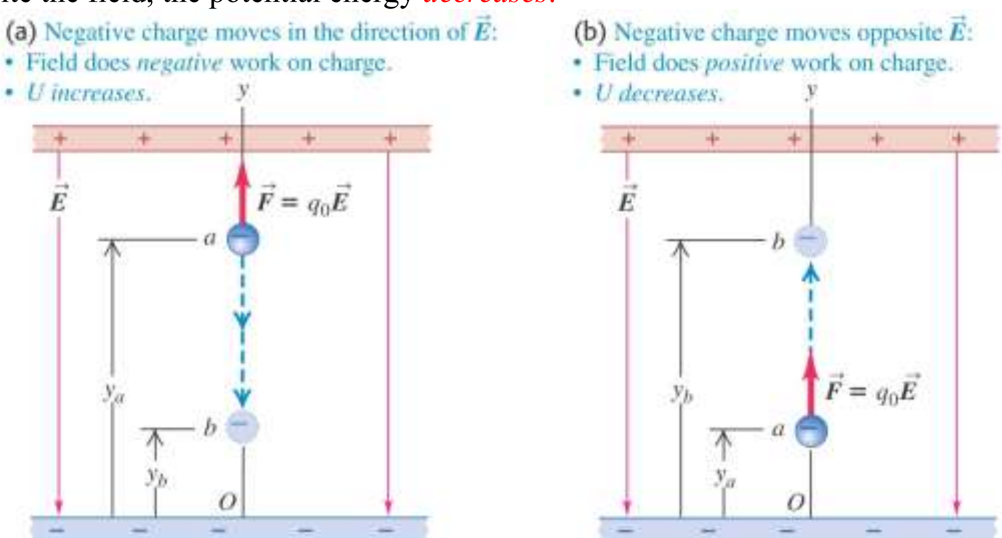
A positive charge moving in a uniform field

- If the positive charge moves in the direction of the field, the potential energy *decreases*, but if the charge moves opposite the field, the potential energy *increases*.



A negative charge moving in a uniform field

- If the negative charge moves in the direction of the field, the potential energy *increases*, but if the charge moves opposite the field, the potential energy *decreases*.



Example: A point charge $q = 8.\times 10^{-9}\ C$ is raised $5\ mm$ above the negative plate of a parallel plate capacitor that has an electric field intensity $E = 4 \times 10^4 N/C$.

(a) Find the potential energy of the point charge at this location.

(b) Is the potential energy increasing or decreasing and why?

Solution: (a) The potential energy of the point charge is:

$$U = q\,E\,y$$

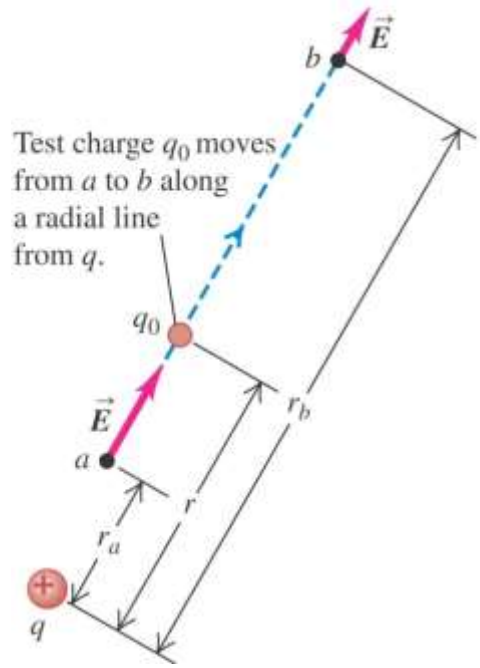
$$= (8.\ \times 10^{-9})(4.\ \times 10^{4})(5.\ \times 10^{-3})$$

$$= 1.60 \times 10^{-6}\,J$$

(b) The potential energy is increasing since the charge is moving opposite the direction of the field.

Electric potential energy of two point charges

Consider a point charge q that sets up an electric field in space. Now a test charge q_0 is placed at position a a distance r_a from q_0. Then q_0 moves to position b a distance r_b from q_0.



What is the change in the potential energy?

The change in potential energy is the negative of the work done to move the test charge from a to b.

The force on the test charge is given by Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}$$

- The work done is force times distance. But the force changes as q_0 moves away from q.
- The force is not constant during the displacement, and we have to integrate to calculate the work $W_{a\to b}$ done on q_0 by this force as q_0 moves from a to b:

Use $dW = F_r dr$

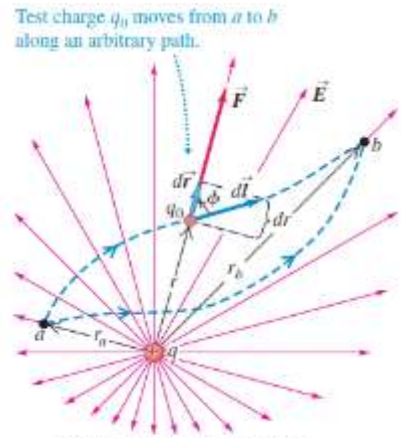
$$\therefore\ W_{a\to b} = \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\,dr = \frac{q q_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

The work done on q_0 by electric field does not depend on path taken, but only on distances r_a and r_b (initial and end points).

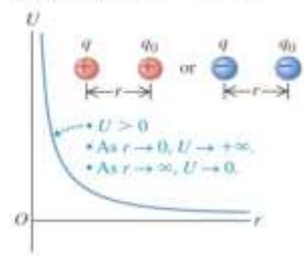
- ❖ let's consider a more general displacement in which a and b do not lie on the same radial line. The work done on q_0 during this displacement is given by

$$W_{a\to b} = \int_{r_a}^{r_b} F.\cos\varphi\ dl = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\cos\varphi\ dl$$

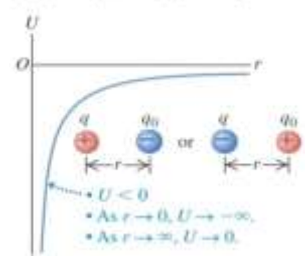
if q_0 returns to its starting point a by a different path, the total work done in the round-trip displacement is zero



(a) q and q_0 have the same sign.



(b) q and q_0 have opposite signs.



- The electric potential energy U when the test charge q_0 is at any distance r from charge q is

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

- The sign of the potential energy depends on the signs of the two charges.
- $U = 0$ when q and q_0 are infinitely apart ($r \to \infty$).

The change in potential energy, is the negative of this work.

$$\Delta U = -W = U_b - U_a = -\frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

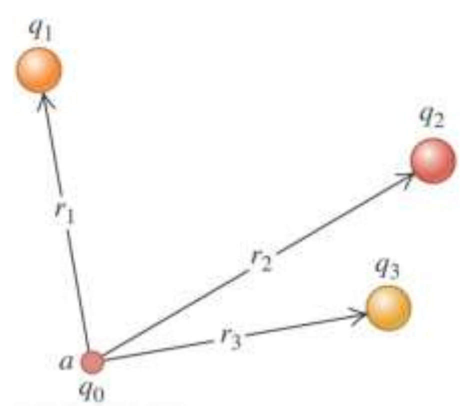
Electrical potential with several point charges

- The potential energy associated with q_0 depends on the other charges and their distances from q_0.

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- The total potential energy associated with a system of multiple charges is

$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_iq_j}{r_{ij}}$



Example: Two point charges are located on the x-axis, $q_1 = -e$ at $x = 0$ and $q_2 = +e$ at $x = a$.

(a) Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$.

(b) Find the total potential energy of the system of the three charges.

Solution: (a) The work W equals the difference between the potential energy U associated with q_3 when it is at $x = 2a$ and the potential energy $U_\infty = 0$ when it is infinitely far away. So the work required is equal to U

The distances between the charges are $r_{13} = 2a$ and $r_{23} = a$

$$\therefore W = U = \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) = \frac{+e}{4\pi\epsilon_0}\left(\frac{-e}{2a} + \frac{+e}{a}\right) = \frac{+e^2}{8\pi\epsilon_0 a}$$

(b) the total potential energy of the three charge system is

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(-e)(e)}{a} + \frac{(-e)(e)}{2a} + \frac{(e)(e)}{a}\right] = \frac{-e^2}{8\pi\epsilon_0 a}$$

Electric potential

DEFINITION: ***Electrical Potential is Potential Energy per Unit Charge***

$$Electric\ Potential(V) = \frac{Potential\ Energy(U)}{Unit\ Charge(q_0)}$$

Units: $\boldsymbol{Volt\ (V) = J/C = Nm/C}$

Potential energy and charge are both scalars, so potential is a scalar quantity.

Therefore divide all terms by q_0

$$\frac{W_{a\to b}}{q_0} = \frac{U_b}{q_0} - \frac{U_a}{q_0} = \frac{-\frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}{q_0}$$

Thus $V_b - V_a = -\frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$

Or $V_a - V_b = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = V_{ab}$

The difference $V_{ab} = V_a - V_b$ *is called the potential of a with respect to b. or the potential difference between a and b*

The potential of *a* with respect to *b* ($V_{ab} = V_a - V_b$) equals:

- ✓ the work done by the electric force when a *unit* charge moves from *a* to *b*.
- ✓ the work that must be done to move a *unit* charge slowly from *b* to *a* against the electric force.

- ➢ Potential due to a point charge *q*

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- ➢ Potential due to a collection of point charge

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- ➢ Potential due to a continuous distribution of charge

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

Finding electric potential from the electric field

The force $\boldsymbol{F}$ on a test charge q_0 can be written as $\boldsymbol{F} = \boldsymbol{q_0 E}$. The work done by the electric force as the test charge moves from a to b is given by

$$W_{a\to b} = \int_a^b \vec{F}.\vec{dl} = \int_a^b q_0\vec{E}.\vec{dl}$$

If we divide this by q_0, the result is

$$V_a - V_b = \int_a^b \vec{E}\cdot\vec{dl} = \int_a^b E\cos\phi\, dl \quad \text{(potential difference as an integral of } \vec{E}\text{)}$$

ELECTRON VOLT

Definition: An electron volt is a unit for energy. It is the work necessary to move an electron (charge $e = 1.6\times10^{-19}\ C$) a potential difference of $1\ volt$.

The work to move a charge across a potential difference is

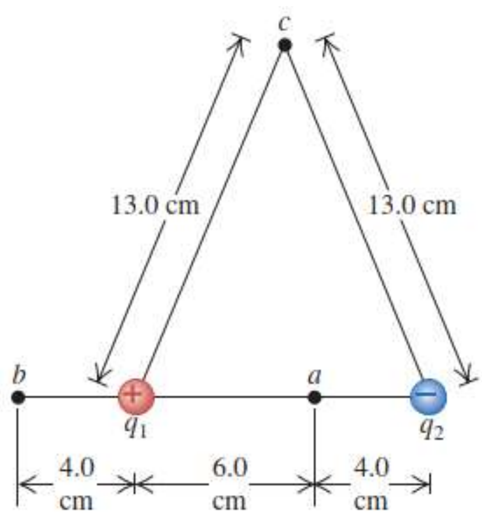
$$W = qV = (1.6\times10^{19}C)(1\ V) = 1.6\times10^{-19}J$$

Therefore,

$$1\ eV = 1.6\times10^{-19}J$$

Example: (Potential due to two point charges)

An electric dipole consists of point charges $q_1 = +12\ nC$ and $q_2 = -12\ nC$ placed $10\ cm$ apart. Compute the electric potentials at point $a,\ b,\ and\ c$. Compute the potential energy associated with a $+4\ nC$ point charge if it is placed in $a,\ b,\ and\ c$.



Solution: At point a: $r_1 = 0.06\ m$ and $r_2 = 0.04\ m$, so the potential at a is

$$V_a = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

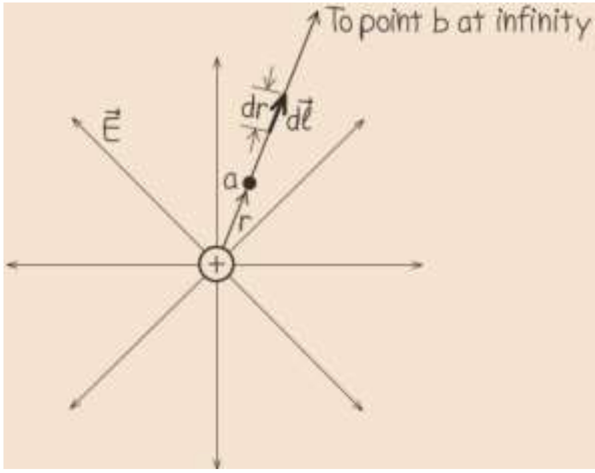
$$= 9\times10^9\left(\frac{12\times10^{-9}}{0.06} + \frac{-12\times10^{-9}}{0.04}\right)$$

$$= -900\ V$$

In a similar way you can show that the potential at point b (where $r_1 = 0.04m$ and $r_2 = 0.14m$) is $V_b = 1930\ V$ and that the potential at point c (where $r_1 = r_2 = 0.13m$) is $V_c = 0$.

Finding potential by integration

Example: Find the potential at a distance r from a point charge q by integrating the electric field.



Solution: The most convenient path is a radial line as shown in Figure, so that $d\boldsymbol{l}$ is in the radial direction and has magnitude dr. Writing $d\boldsymbol{l} = \boldsymbol{r}dr$

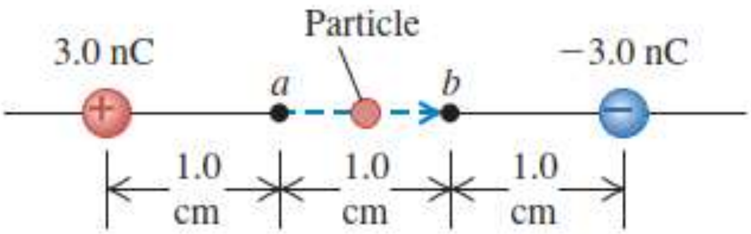
$$V - 0 = V = \int_r^\infty \boldsymbol{E}\cdot d\boldsymbol{l}$$

$$= \int_r^\infty \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr = \frac{q}{4\pi\epsilon_0 r}\bigg|_r^\infty = 0 - \left(\frac{q}{4\pi\epsilon_0 r}\right)$$

$$\therefore V = \frac{q}{4\pi\epsilon_0 r}$$

Moving through a potential difference

Example: a dust particle with mass $m = 5\times10^{-9}\ kg$ and charge $q_0 = 2\ nC$ starts from rest and moves in a straight line from point a to point b as shown What is its speed v at point ?



Solution: Only the conservative electric force acts on the particle, so mechanical energy is conserved: $K_a + U_a = K_b + U_b$,

$$K_a = 0\ (particle\ starts\ from\ rest)$$

$U = q_0V,\quad V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

$\therefore\ q_0V_a = \frac{1}{2}mv^2 + q_0V_b$

$$\therefore v = \sqrt{\frac{2q_0(V_a - V_b)}{m}},$$

$$\therefore V_a = (9\times10^9)\left(\frac{3\times10^{-9}}{0.01} + \frac{(-3\times10^{-9})}{0.02}\right) = 1350\ V$$

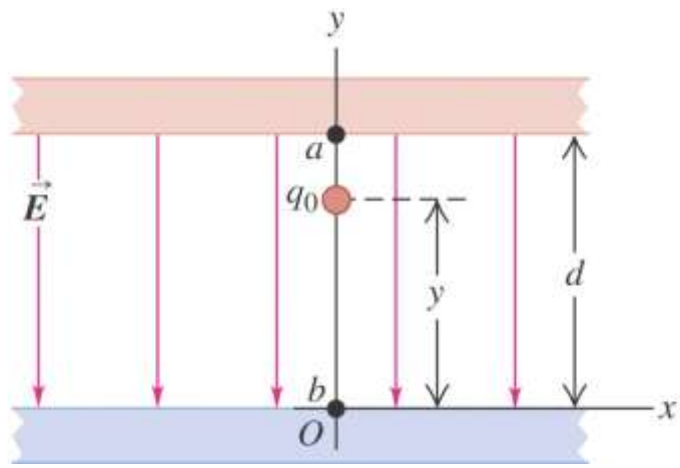
$$V_b = (9\times10^9)\left(\frac{3\times10^{-9}}{0.02} + \frac{(-3\times10^{-9})}{0.01}\right) = -1350\ V$$

$$V_a - V_b = 1350 - (-1350) = 2700\ V$$

$$\therefore v = \sqrt{\frac{2(2\times10^{-9})(2700)}{5\times10^{-9}}} = 46\ m/s$$

Oppositely charged parallel plates

Example: Find the potential at any height y between the two oppositely charged parallel plates.



Solution

The potential $V(y)$ at coordinate y is the potential energy per unit charge:

$$V(y) = \frac{U(y)}{q_o} = \frac{q_o E y}{q_o} = Ey$$

The potential decreases as we move in the direction of from the upper to the lower plate. At point a, where $y = d$ and $V(y) = V_a$,

$$V_a - V_b = Ed - E0 = Ed$$

$$V_{ab} = Ed$$

$$\therefore E = \frac{V_{ab}}{d},$$

where V_{ab} is the potential of the positive plate with respect to the negative plate. That is, the electric field equals the potential difference between the plates divided by the distance between them. For a given potential difference the smaller the distance between the two plates, the greater the magnitude of the electric field.

An infinite line charge or conducting cylinder

Example: Find the potential at a distance r from a very long line of charge with linear charge density λ.

Solution: the electric field at a radial distance r from a long straight-line charge has only a radial component given by $E_r = \lambda / 4\pi\epsilon_0 r$. We use this expression to find the potential by integrating $\boldsymbol{E}$.

Since the field has only a radial component, we have $\boldsymbol{E} \cdot d\boldsymbol{l} = E_r dr$. Hence from the potential of any point a with respect to any other point b at radial distances r_a and r_b from the line of charge, is

$$V_a - V_b = \int_a^b \boldsymbol{E} \cdot d\boldsymbol{l} = \int_{\boldsymbol{a}}^{\boldsymbol{b}} E_r dr = \frac{\lambda}{2\pi\epsilon_0} \int_{r_a}^{r_b} \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0} ln \frac{r_b}{r_a}$$

We set $V_b = 0$ at point at an arbitrary, but finite radial distance r_0. Then the potential $V = V_a$ at point a at a radial distance r is given

$$V - 0 = \frac{\lambda}{2\pi\epsilon_0} ln \frac{r_0}{r}$$

If we choose r_0 to be the radius of the cylinder R, so that $V = 0$ at $r = R$, then at any point a for which $r > R$

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{R}{r}$$

Inside the cylinder $E = 0$, and V has the same value (zero) as on the cylinder's surface.

A ring of charge

Example: Electric charge Q is distributed uniformly around a thin ring of radius a. Find the potential at a point P on the ring axis at a distance x from the center.

Solution: the distance from each charge element dq to P is $r = \sqrt{x^2 + a^2}$.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+a^2}}\int dq$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$

Potential of a line of charge

Positive electric charge Q is distributed uniformly along a line of length $2a$ lying along the y-axis between $y = -a$ and $= +a$. Find the electric potential at a point P on the x-axis at a distance x from the origin.

Solution: the element of charge dQ corresponding to an element of length dy on the rod is $dQ = (Q/2a)dy$. The distance from dQ to P is $\sqrt{x^2 + y^2}$, so

$$dV = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{\sqrt{x^2+y^2}}$$

$$\therefore V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{dy}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\left(\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}\right)$$

Lecture 5: Capacitance and dielectric

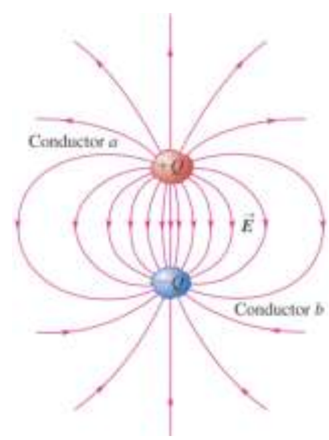
Capacitance and Dielectrics

- ***Capacitors and capacitance***
  - Any two conductors separated by an insulator form a ***capacitor.***
  - The definition of capacitance is

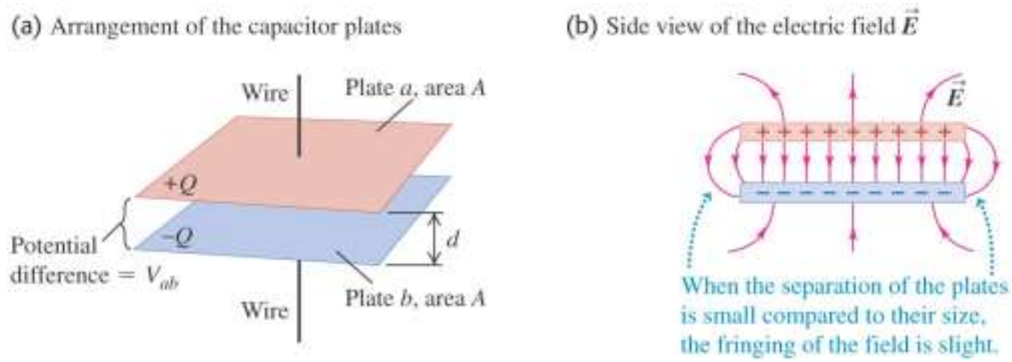
$$C = \frac{Q}{V_{ab}}$$

  - The SI unit of capacitance is called one farad (1 F),
    One farad is equal to one coulomb per volt

$$F = {}^{C}/_{V}$$

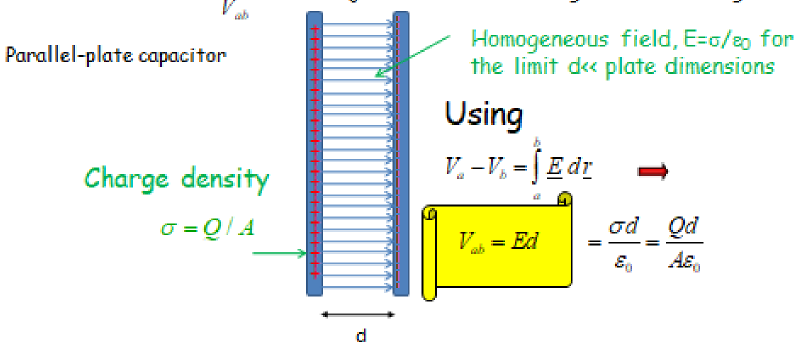


- ***Parallel-plate capacitor***
  - A *parallel-plate capacitor* consists of two parallel conducting plates separated by a distance that is small compared to their dimensions.



Calculating capacitance

According to $C = \frac{Q}{V_{ab}}$ the major task in calculating C is calculating V_{ab}



Using

$$V_a - V_b = \int_a^b \underline{E}\, d\underline{r} \Rightarrow V_{ab} = Ed = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{A\varepsilon_0}$$

⇒ We obtain the capacitance C of a parallel-plate capacitor in vacuum as $C = \varepsilon_0 A/d.$

Note: 1F is a huge capacitance. More typical values are $1\mu F=10^{-6}F$ to $1pF=10^{-12}F$

Example1: The parallel plates of a $1F$ capacitor are $1\ mm$ apart. What is their area?

Solution: $C = \varepsilon_0 A/d.$

$$\therefore A = \frac{Cd}{\epsilon_0} = \frac{(1)(1\times10^{-3})}{8.85\times10^{-12}} = 1.1\times10^{8}\ m^2$$

Example 2: The plates of a parallel-plate capacitor in vacuum are $5\ mm$ apart and in $2\ m^2$ area. A 10kV potential difference is applied across the capacitor.

Compute (a) the capacitance;

(b) The charge on each plate; and

(c) The magnitude of the electric field between the plates.

Solution: (a) $C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12})(2)}{5\times10^{-3}} = 3.54\times10^{-9}F = 3.54\ nF$

(b) The charge on the capacitor is

$$Q = CV_{ab} = (3.54\times10^{-9})(1\times10^{4}) = 3.54\times10^{-5} = 35.4\mu C$$

The plate at higher potential has charge $+35.4\mu C$, and the other

plate has charge $-35.4\mu C$.

(c) The magnitude of the electric field is

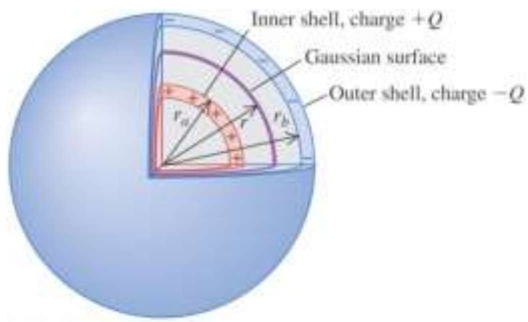
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_{0A}} = \frac{3.54 \times 10^{-9}}{8.85 \times 10^{-12} \times 2} = 2 \times 10^{6} N/C$$

Note: the dimension of $\epsilon_0 = \frac{Cd}{A} = \left[\frac{Farad}{meter}\right]$

$$\epsilon_0 = \frac{Cd}{A} = \frac{\frac{Q}{V_{ab}}d}{A} = \frac{Qd}{AV_{ab}} = \frac{Qd}{AEd} = \frac{Q}{A\frac{F}{Q}} = \frac{Q^2}{AF} = \left[\frac{Coulomb^2}{m^2.Newton}\right]$$

A spherical capacitor

Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge $+Q$ and outer radius r_a and the outer shell has charge $-Q$ and inner radius r_b. Find the capacitance of this spherical capacitor.



Solution: The potential at any point between the spheres is $V = Q/4\pi\epsilon_0 r$. Hence the potential of the inner (positive) conductor at $r = r_a$ with respect to that of the outer (negative) conductor at $r = r_b$ is

$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b}$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r_b - r_a}{r_a r_b}$$

The capacitance is then

$$C = \frac{Q}{V_{ab}} = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

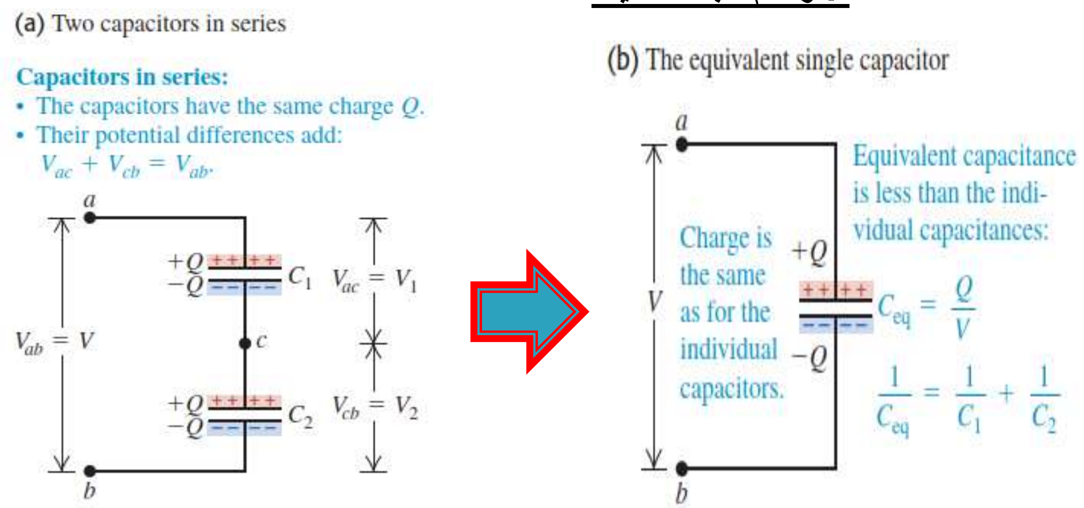
As an example, if $r_a = 9.5$ cm and $r_b = 10.5$ cm,

$$C = 4\pi(8.85 \times 10^{-12}\ \text{F/m})\frac{(0.095\ \text{m})(0.105\ \text{m})}{0.010\ \text{m}}$$

$$= 1.1 \times 10^{-10}\ \text{F} = 110\ \text{pF}$$

Capacitors in series

Capacitors are in *series* if they are connected one after the other.



❖ In a **series** connection the magnitude of ***charge*** on all plates is ***the same***.

$C = \frac{Q}{V}$ ⟶ $V = \frac{Q}{C}$

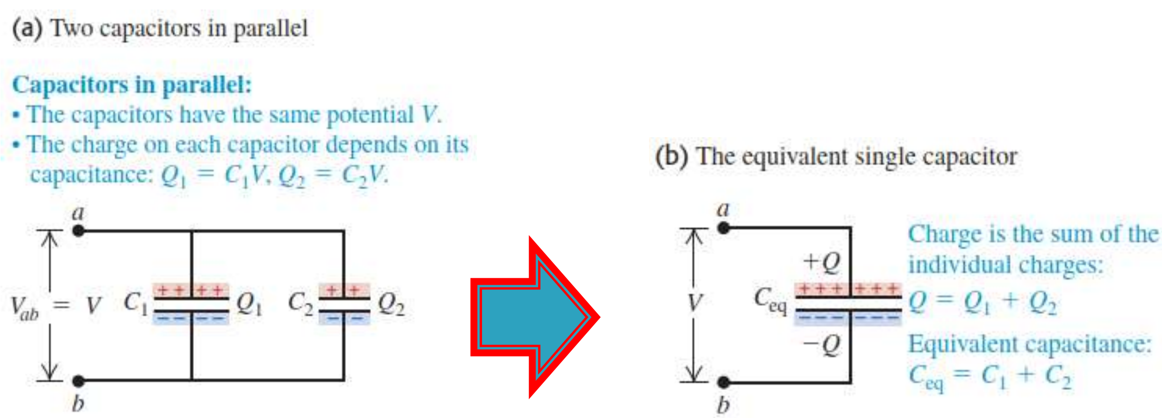
$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$ ⟶ $\frac{V}{Q} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{1}{C_{eq}}$

The *equivalent capacitance* of a series combination is given by:

$$\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in parallel

Capacitors are connected in *parallel* between a and b if the potential difference is V_{ab} the *same* for all the capacitors. b



Potential difference V_{ab} is the *same* for all the capacitors.

$C = \frac{Q}{V}$ ⟶ $Q = CV$

$Q = Q_1 + Q_2 = C_1V + C_2V = V(C_1 + C_2)$ ⟶ $\frac{Q}{V} = C_1 + C_2 = C_{eq}$

The *equivalent capacitance* of a *Parallel* combination is given by:

$$C_{eq} = C_1 + C_2$$

Example 1: let $C_1 = 6\mu F$ and $C_2 = 3\mu F$. Find the equivalent capacitance $V_{ab} = 18\ V$ and the charge and potential difference for each capacitor when the capacitors are connected
(a) in series and (b) in parallel.

Solution: (a) for a series combination,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6\times 10^{-6}} + \frac{1}{3\times 10^{-6}} = \frac{3}{6\times 10^{-6}}$$

$C_{eq} = 2\times 10^{-6} = 2\mu F$

The charge on each capacitor in series is the same as that on the equivalent capacitor:

$Q = C_{eq}V = 2\times 10^{-6}\times 18 = 36\times 10^{-6} = 36\mu C.$

The potential difference across each capacitor is inversely proportional to its capacitance:

$$V_1 = \frac{Q}{C_1} = \frac{36\times 10^{-6}}{6\times 10^{-6}} = 6V$$

$$V_2 = \frac{Q}{C_2} = \frac{36\times 10^{-6}}{3\times 10^{-6}} = 12V$$

(b) For a parallel combination,

$$C_{eq} = C_1 + C_2 = 6\mu F + 3\mu F = 9\mu F$$

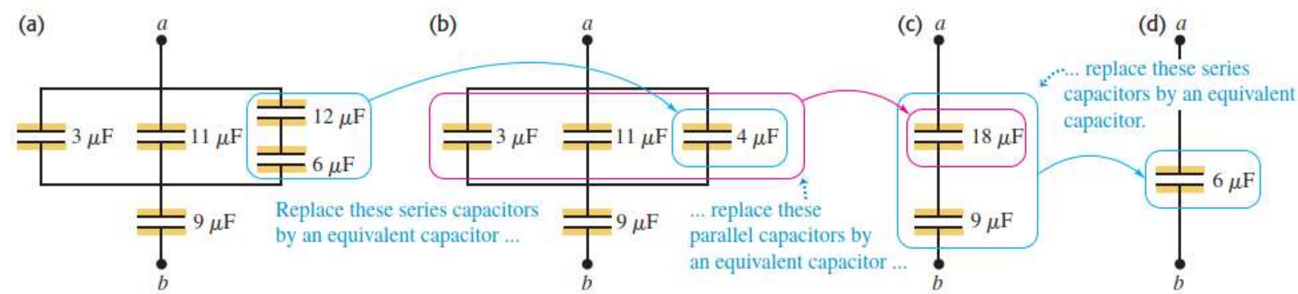
The potential difference across each of the capacitors is the same as that across the equivalent capacitor, 18 V.
The charge on each capacitor is directly proportional to its capacitance:

$$Q_1 = C_1V = 6\times 10^{-6}\times 18 = 108\mu C$$

$$Q_2 = C_2V = 3\times 10^{-6}\times 18 = 54\mu C$$

Example 2: Find the equivalent capacitance of the five-capacitor network shown in the Figure.

24.10 (a) A capacitor network between points a and b. (b) The 12-μF and 6-μF capacitors in series in (a) are replaced by an equivalent 4-μF capacitor. (c) The 3-μF, 11-μF, and 4-μF capacitors in parallel in (b) are replaced by an equivalent 18-μF capacitor. (d) Finally, the 18-μF and 9-μF capacitors in series in (c) are replaced by an equivalent 6-μF capacitor.



$$\frac{1}{C'} = \frac{1}{12\ \mu F} + \frac{1}{6\ \mu F} \qquad C' = 4\ \mu F$$

$$C'' = 3\ \mu F + 11\ \mu F + 4\ \mu F = 18\ \mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{18\ \mu F} + \frac{1}{9\ \mu F} \qquad C_{eq} = 6\ \mu F$$

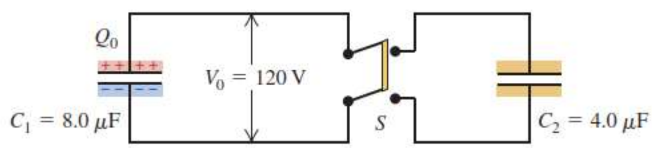
د. وسام عبدالله لطيف

Energy stored in a capacitor

The potential energy stored in a capacitor is

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Example: We connect a capacitor $C_1 = 8\mu F$ to a power supply, charge it to a potential difference $V_0 = 120V$, and disconnect the power supply. Switch is open. (a) What is the charge Q_0 on C_1?
(b) What is the energy stored in C_1?
(c) Capacitor $C_2 = 4\mu F$ is initially uncharged. We close switch S. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor?
(d) What is the final energy of the system?



Solution: (a) the initial charge Q_0 on C_1 is

$$Q_0 = C_1V_0 = 8 \times 10^{-6} \times 120 = 960\ \mu C$$

(b) The energy initially stored in C_1 is

$$U_{initial} = \frac{1}{2}Q_0V_0 = \frac{1}{2} \times 960 \times 10^{-6} \times 120 = 0.058\ \text{J}$$

(c) The charge Q_0 is distributed over the two capacitors.

$$Q_0 = Q_1 + Q_2$$

Since the two capacitors are connected in parallel, V is the same for both.

$$V = \frac{Q_0}{C_1 + C_2} = \frac{960\mu C}{8\mu F + 4\mu F} = 80\ V$$

(d) The final energy of the system is

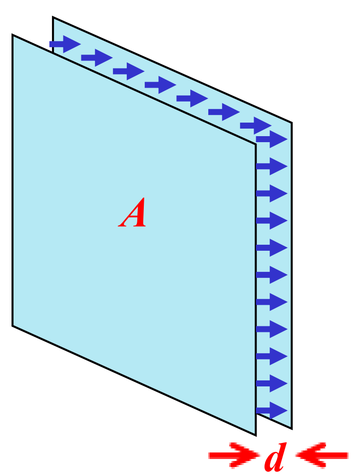
$$U_{final} = \frac{1}{2}Q_1V + \frac{1}{2}Q_2 + \frac{1}{2}Q_0$$

$$= \frac{1}{2}(960 \times 10^{-6})(80) = 0.038\ J$$

Electric-Field Energy

We can charge a capacitor by moving electrons directly from one plate to another. This requires doing work against the electric field between the plates. Thus we can think of the energy as being stored in the field in the region between the plates. To develop this relationship, let's find the *energy per unit volume* in the space between the plates of a parallel-plate capacitor with plate area A and separation d. We call this the ***energy density***, denoted by u,

$$\textbf{energy density} = u = \frac{\textbf{stored potential energy}}{\textbf{Volume}} = \frac{\frac{1}{2}CV^2}{Ad}$$



The capacitance is $C = \epsilon_0 \frac{A}{d}$ and the potential is given by $V = Ed$
Therefore,

$$u = \frac{\frac{1}{2}(\epsilon_0 \frac{A}{d})(E^2d^2)}{Ad} = \frac{1}{2}\epsilon_0E^2$$

$$= \frac{1}{2}(960 \times 10^{-6})(80) = 0.038\ J$$

Example:

(a) What is the magnitude of the electric field required to store 1 J of electric potential energy in a volume of $1m^3$ in vacuum?

(b) If the field magnitude is 10 times larger than that, how much energy is stored per cubic meter?

Solution: (a)

$$u = \frac{1}{2}\epsilon_0 E^2$$

$$E = \sqrt{\frac{2u}{\epsilon_0}} = \sqrt{\frac{2}{8.85 \times 10^{-12}}} = 4.75 \times 10^5 N/C$$

(b) Since u is proportional to E^2, so if E increases by a factor of 10 then u increases by a factor of 10^2. So the energy density becomes $u = 100\ J/m^3$

Dielectrics

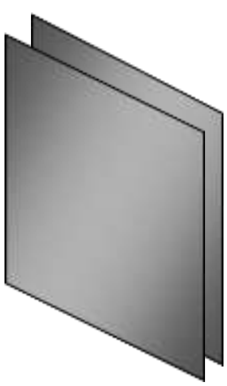
A *dielectric* is *a non-conducting material that, when placed between the plates of a capacitor, increases the capacitance.*

- ❖ Dielectrics include rubber, plastic, and waxed paper
- ➢ If the dielectric completely fills the space between the plates, the capacitance increases by a factor K called the dielectric constant

$$K = {}^{C}/_{C_0}$$

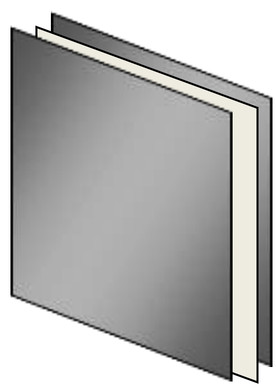
No dielectric

$$C_0 = \frac{Q}{V_0}$$



with dielectric

$$C = \frac{Q}{V}$$



$$Q = C_0 V_0 = CV \;\Rightarrow\; \frac{V_0}{V} = \frac{C}{C_0} = K \;\Rightarrow\; \therefore V = \frac{V_0}{k}$$

Since $K \geq 1$ ⇒ $V < V_0$ ⇒ $C > C_0$

- ➢ The capacitance when the dielectric is present between two plates of area A and d apart is given by

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

$$\epsilon = K\epsilon_0$$

Example: Suppose the parallel plates each have an area of ($0.2m^2$) and are ($1cm$) apart. We connect the capacitor to a power supply, charge it to a potential difference $V_0 = 3\ kV$ and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to $1\ kV$, while the charge on each capacitor plate remains constant. Find

(a) The original capacitance.
(b) The magnitude of charge on each plate;
(c) The capacitance after the dielectric is inserted;
(d) The dielectric constant of the dielectric;
(e) The permittivity ϵ of the dielectric;
(f) The original electric field E_0 between the plates; and
(g) The electric field E after the dielectric is inserted.

Solution: (a) With vacuum between the plates, $K = 1$

$$C_0 = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}) \frac{0.2}{0.01} = 1.77 \times 10^{-10}$$

(b) $Q = C_0 V_0 = 1.77 \times 10^{-10} \times 3 \times 10^3 = 5.31 \times 10^{-7}\ C$

(c) When the dielectric is inserted, Q is unchanged but the potential difference decreases to $V = 1\ kV$. Hence, the new capacitance is

$$C = \frac{Q}{V} = \frac{5.31 \times 10^{-7}}{1 \times 10^3} = 5.31 \times 10^{-10}\ F$$

(d) The dielectric constant is

$$K = \frac{C}{C_0} = \frac{5.31 \times 10^{-10}}{1.77 \times 10^{-10}} = 3 \qquad \text{Or} \quad K = \frac{V_0}{V} = \frac{3000}{1000} = 3$$

(e) The permittivity of the dielectric is

$$\epsilon = K\epsilon_0 = 3 \times 8.85 \times 10^{-12}$$
$$= 2.66 \times 10^{-11} C^2/Nm^2$$

(f) Since the electric field between the plates is uniform, its magnitude is the potential difference divided by the plate separation:

$$E_0 = \frac{V_0}{d} = \frac{3000}{0.01} = 3 \times 10^5\ V/m$$

(g) After the dielectric is inserted,

$$E = \frac{V}{d} = \frac{1000}{0.01} = 1 \times 10^5\ V/m$$

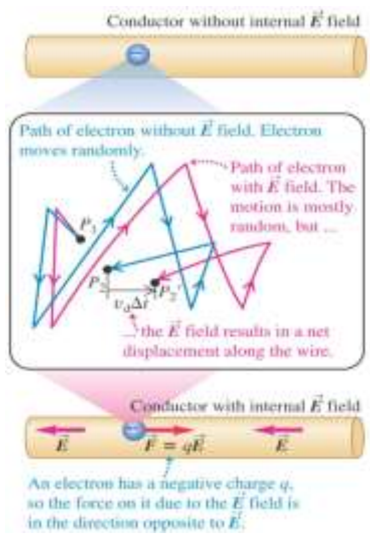
Lecture 6: Current, resistance, and electromotive force

CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

Current

A current is any motion of charge from one region to another. Current is defined as

$$I = \frac{dQ}{dt}$$



In electrostatic situations the electric field is zero everywhere inside the conductor, and there is no current. However, this does not mean that all charges within the conductor are at rest. In an ordinary metal such as copper or aluminum, some of the electrons are free to move within the conducting material. These free electrons move randomly in all directions, somewhat like the molecules of a gas but with much greater speeds, of the order of $10^6 m/s$. The electrons nonetheless do not escape from the conducting material, because they are attracted to the positive ions of the material. The motion of the electrons is random, so there is no net flow of charge in any direction and hence no current.

- An electric field in a conductor causes charges to flow.

Now consider what happens if a constant, steady electric field E **is** established inside a conductor. A charged particle (such as a free electron) inside the conducting material is then subjected to a steady force $\boldsymbol{F} = q\boldsymbol{E}$. If the charged particle were moving in vacuum, this steady force would cause a steady acceleration in the direction of $\boldsymbol{F}$ and after a time the charged particle would be moving in that direction at high speed. But a charged particle moving in a conductor undergoes frequent collisions with the massive, nearly stationary ions of the material. In each such collision the particle's direction of motion undergoes a random change. The net effect of the electric field $\boldsymbol{E}$ is that in addition to the random motion of the charged particles within the conductor, there is also a very slow net motion or *drift* of the moving charged particles as a group in the direction of the electric force $\boldsymbol{F} = q\boldsymbol{E}$. This motion is described in terms of the ***drift velocity*** $\boldsymbol{v_d}$ of the particles. As a result, there is a net current in the conductor.

The current through the cross-sectional area A is defined as the net charge flowing through the area per unit time. Thus, if a net charge dQ flows through an area in a time dt , the current through the area is

$$I = \frac{dQ}{dt}$$

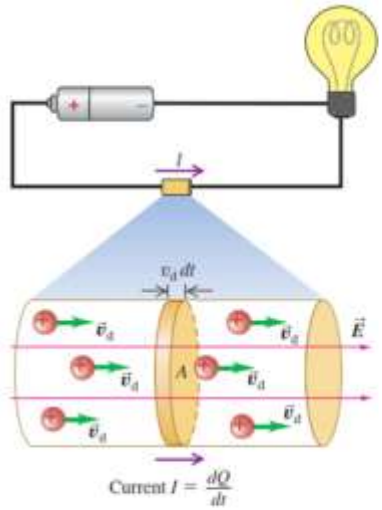
The SI unit of current is the ampere; *one ampere is defined to be one coulomb per second.* $1A = 1C/s$

Current, drift velocity, and current density

Suppose there are n moving charged particles per unit volume. We call n the c**oncentration** of particles; its SI unit is m^{-3}.

Assume that all the particles move with the same drift velocity with magnitude v_d
In a time interval dt , each particle moves a distance $v_d dt$ The particles that flow out of the right end of the shaded cylinder with length $v_d dt$ during
dt are the particles that were within this cylinder at the beginning of the interval dt . The volume of the cylinder is $Av_d\ dt$, and the number of particles within it is $nAv_d\ dt$. If each particle has a charge q, the charge dQ that flowsout of the end of the cylinder during time dt is

$$dQ = q(nAv_d dt)$$



And the current is

$$I = \frac{dQ}{dt} = nqv_d A$$

The current per unit cross-sectional area is called the *current density J*,

$$J = \frac{I}{A} = nqv_d$$

The SI units of current density are amperes per square meter (A/m^2).

If the moving charges are negative, the drift velocity is opposite to $\boldsymbol{E}$ but the current is still in the same direction as $\boldsymbol{E}$ at each point in the conductor. Hence, the current I and the current density J do not depend on the sign of the charge, and so we use the absolute value of the charge$|q|$:

$$I = \frac{dQ}{dt} = n|q|v_d A$$

$$J = \frac{I}{A} = n|q|v_d$$

Example: An 18-gauge copper wire (the size usually used for lamp cords), with a diameter of $1.02\ mm$ carries a constant current of $1.67\ A$ to a 200-W lamp. The free-electron density in the wire is 8.5×10^{28} per cubic meter. Find

(a) The current density and
(b) The drift speed.

Solution: (a) The cross-sectional area is

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.02 \times 10^{-3})^2}{4} = 8.17 \times 10^{-7} m^2$$

The magnitude of the current density is

$$J = \frac{I}{A} = \frac{1.67}{8.17 \times 10^{-7}} = 2.04 \times 10^6\ A/m^2$$

(b) The drift velocity

$$v_d = \frac{J}{n|q|} = \frac{2.04 \times 10^6}{(8.5 \times 10^{28})|-1.6 \times 10^{-19}|} = 1.5 \times 10^{-4} m/s = 0.15 mm/s$$

❖ Resistivity

- The *resistivity* (ρ) of a material is the ratio of the electric field in the material to the current density it causes:

$$\rho = \frac{E}{J}$$

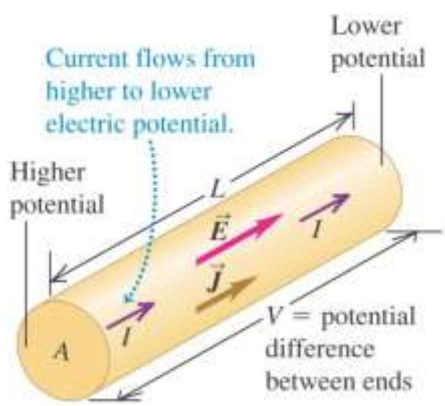
- The units of ρ is $\frac{(V/m)}{(A/m^2)} = V.m/A$
  - A perfect conductor would have zero resistivity, and a perfect insulator would have an infinite resistivity. Metals and alloys have the smallest resistivity and are the best conductors.
  - Semiconductors have resistivity intermediate between those of metals and those of insulators. These materials are important because of the way their resistivity is affected by temperature and by small amounts of impurities.
- The *conductivity*(σ) is the reciprocal of the resistivity. $\sigma = \frac{1}{\rho}$
  - The resistivity of a metallic conductor nearly always increases with increasing temperature.
  - As temperature increases, the ions of the conductor vibrate with greater amplitude, making it more likely that a moving electron will collide with an ion, this impedes the drift of electrons through the conductor and hence reduces the current.
  - Over a small temperature range (up to 100^0 or so), the resistivity of a metal can be represented approximately by the equation

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

where ρ_0 is the resistivity at T_0 a reference temperature (often taken as 0^0C or 20^0C) and $\rho(T)$ is the resistivity at temperature T, which may be higher or lower than T_0The factor α is called the *temperature coefficient of resistivity.*

❖ Resistance

- For a conductor of length L and cross-sectional area A, the potential difference between its ends is V.
- the electric field is $E = \frac{V}{L}$ and
- the current density $J = \frac{I}{A}$
- but $E = \rho J$



$$\therefore \frac{V}{L} = \rho \frac{I}{A}\text{ , so } V = \frac{\rho L}{A} I$$

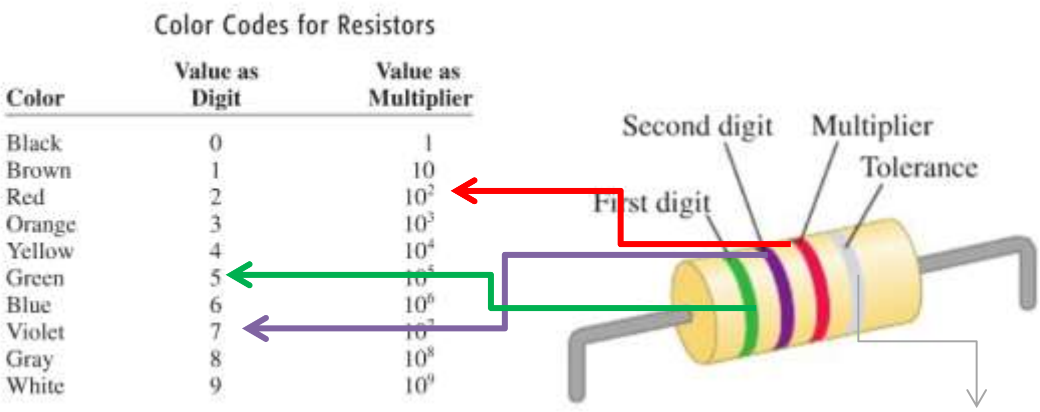
The constant of proportionality between V and I is the <u>*resistance*</u> R

$$\therefore R = \frac{\rho L}{A}$$

➢ The equation $V = IR$ is called ***Ohm's law***.

❖ *Color code for resistors and symbols in circuit diagrams*

❖ This resistor has a resistance of 5.7 kΩ with a tolerance of ±10%.



Color Codes for Resistors

Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

$$R = 5700 \quad \Omega = 5.7\ \text{k}\Omega \pm 10\%$$

Because the resistivity of a material varies with temperature, the resistance of a specific conductor also varies with temperature.

$$R(T) = R_0[1 + \alpha(T - T_0)]$$

Where R_0 is the resistivity at T_0 (often taken as 0^0C or 20^0C) and $R(T)$ is the resistivity at temperature T, the *temperature coefficient of resistance* α *is the same as for the resistivity if L and A do not change appreciably with temperature* .

Example: A copper wire has a cross-sectional area of $8.2 \times 10^{-7}\ m^2$. It carries a current of $1.67\ A$ and of resistivity $1.72 \times 10^{-7}\ \Omega\,.m$. Find

(a) The electric-field magnitude in the wire;

(b) The potential difference between two points in the wire $50\ m$ apart;

(c) The resistance of a length $50\ m$ of this wire.

Solution: (a) the electric field magnitude is

$$E = \rho J = \frac{\rho I}{A} = \frac{1.72 \times 10^{-7} \times 1.67}{8.2 \times 10^{-7}} = 0.035\ V/m$$

(b) The potential difference is

$$V = EL = 0.035 \times 50 = 1.75\ V$$

(c) The resistance of 50 m of this wire is

$$R = \frac{\rho L}{A} = \frac{1.72 \times 10^{-7} \times 50}{8.2 \times 10^{-7}} = 1.05\ \Omega$$

➢ the same result can be found from

$$R = \frac{V}{I} = \frac{1.75}{1.67} = 1.05\ \Omega$$

Example 2: Suppose the resistance of a copper wire is 1.05 Ω at 20^0 Find the resistance at 0^0C *and* $100^0\ C$. The temperature coefficient of copper is $0.00393\ (C)^{-1}$.

Solution:

- the temperature at $T = 0^0C$ is

$$R(0) = R_0[1 + \alpha(T - T_0)]$$
$$= 1.05[1 + (0.00393)(0 - 20)] = 0.97\,\Omega$$

- the temperature at $T = 100^0C$ is

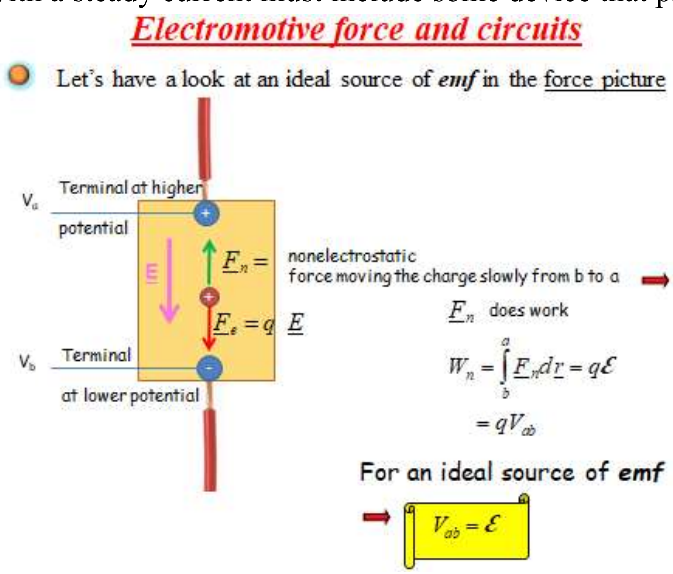
$$R(100) = 1.05[1 + (0.00393)(100 - 20)] = 1.38\,\Omega$$

❖ *Electromotive Force and Circuits*

- In an electric circuit there should be a device that acts like the water pump in a fountain (source of ***emf***.)
- In this device, the charge travels "uphill" from lower to higher V (opposite to normal conductor) due to the ***emf*** force.
  - ❖ ***emf*** (ε) is not a force but energy/unit charge

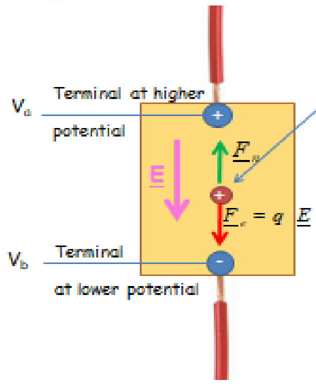
Units: 1 V = 1 J/C

- ***emf*** device convert energy (mechanical, chemical, thermal) into electric potential energy and transfer it to circuit.
- Every complete circuit with a steady current must include some device that provides ***emf***



Internal resistance

- Let's integrate a real source into a complete circuit



Charge moving through real source experiences resistance (a friction force)

$\Rightarrow F_n > F_e \Rightarrow W_n = \int_b^a F_n dr = q\mathcal{E} > qV_{ab}$

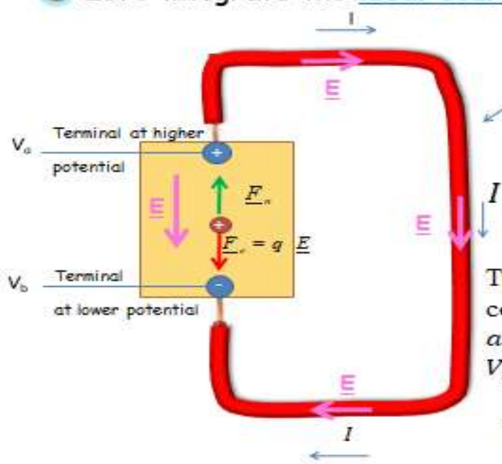
if internal resistance, r, ohmic

$\Rightarrow \mathcal{E} = V_{ab} + Ir$

$\Rightarrow$ Terminal voltage of a source with internal resistance

$$V_{ab} = \mathcal{E} - Ir$$

- Let's integrate the ideal source into a complete circuit



wire with non-zero resistance R

$$\mathcal{E} = V_{ab} = IR$$

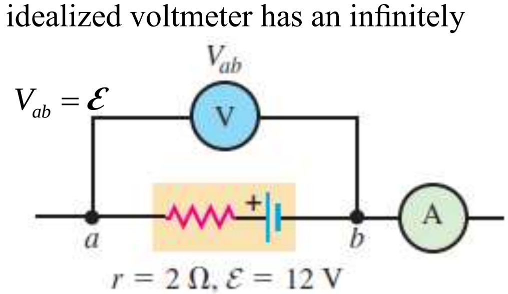
The current in the external circuit connected to the source terminals a and b is still determined by $V_{ab} = IR$

$\varepsilon - Ir = IR$

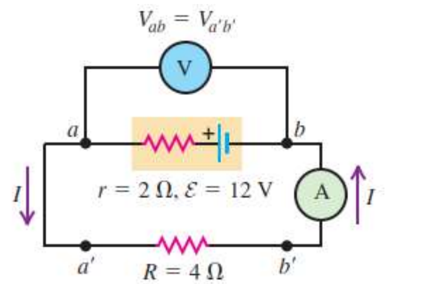
$$\therefore I = \frac{\varepsilon}{R+r}$$

Example 1: The figure shows a source (a battery) with emf and internal resistance $= 2\Omega$. The wires to the left of a and to the right of the ammeter A are not connected to anything. Determine the respective readings V_{ab} and I of the idealized voltmeter V and the idealized ammeter A.

Solution: There is zero current because there is no complete circuit. (Our idealized voltmeter has an infinitely large resistance, so no current flows through it.) Hence the ammeter reads $I = 0$. Because there is no current through the battery, there is no potential difference across its internal resistance. From $V_{ab} = \varepsilon - Ir$ with $I = 0$ the potential difference V_{ab} across the battery terminals is equal to the ***emf.*** So the voltmeter reads $V_{ab} = \varepsilon = 12\ V$. The terminal voltage of a real, nonideal source equals the ***emf*** only if there is no current flowing through the source, as in this example.



Example 2: We add a 4 Ω resistor to the battery in Example 1, to form a complete circuit. What are the voltmeter and ammeter readings V_{ab} and I now?



Solution: The current through the circuit $aa'b'b$ is

$$I = \frac{\varepsilon}{R+r} = \frac{12}{4+2} = 2\,A$$

the idealized ammeter have zero resistance, so there is no potential difference between points a and b or between points a' and b' that is, $V_{ab} = V_{a'b''}$. We find V_{ab} by considering a and b as the terminals of the resistor. From Ohm's law

$$V_{a'b'} = IR = 2(4) = 8\,V$$

we can consider a and b as the terminals of the source. Then,

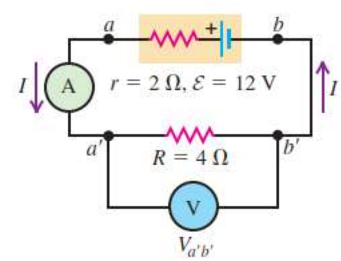
$$V_{ab} = \varepsilon - Ir = 12 - (2 \times 2) = 8\,V$$

Either way, we see that the voltmeter reading is 8 V.

❖ Using voltmeters and ammeters

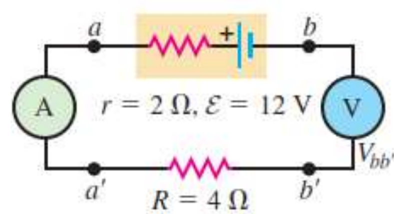
Example 3: We move the voltmeter and ammeter in Example 2 to different positions in the circuit. What are the readings of the ideal voltmeter and ammeter in the situations shown in figure (a) and (b)?

(a)



Solution: (a) The voltmeter now measures the potential difference between points a' and b' As in Example 2, $V_{ab} = V_{a'b}'$, so the voltmeter reads the same as in Example 2 $V_{a'b'} = 8V$.

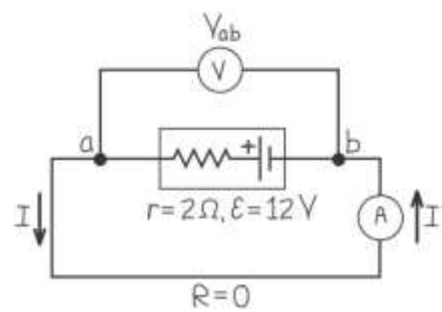
(b)



(b) There is no current through the ideal voltmeter because it has infinitely large resistance. Since the voltmeter is now part of the circuit, there is no current at all in the circuit, and the ammeter reads $I = 0$.

As in Example 1, there is no current, so the terminal voltage equals the ***emf***, and the voltmeter reading is $V_{ab} = \varepsilon = 12\,V$

➢ A source with a short circuit



Example 4: In the circuit of Example 2 we replace the resistor with a zero-resistance conductor. What are the meter readings now?

Solution: since there is no external resistance in the circuit. We must have

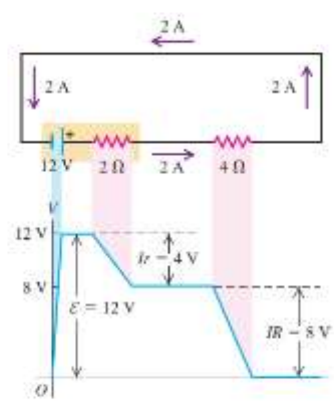
$$V_{ab} = IR = I(0) = 0$$

We can therefore find the current I

$$V_{ab} = \varepsilon - Ir = 0$$

$\therefore I = \frac{\varepsilon}{r} = \frac{12}{2} = 6\,A$

➢ Potential changes around a circuit



The net change in potential energy for a charge q making a round trip around a complete circuit must be *zero*. Hence the net change in potential around the circuit must also be *zero*; in other words, the algebraic sum of the potential differences and ***emf***s around the loop is *zero*.

$$\varepsilon - Ir = IR$$

$$\varepsilon - Ir - IR = 0$$

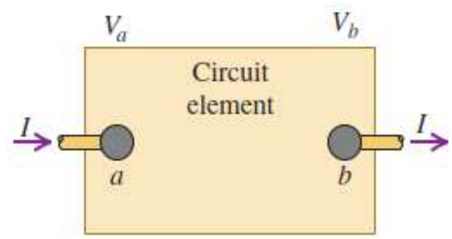
A potential gain of ε is associated with the ***emf***, and potential drops of *Ir* and *IR* are associated with the internal resistance of the source and the external circuit,

If we take the potential to be zero at the negative terminal of the battery, then we have a rise ε and a drop *Ir* in the battery and an additional drop *IR* in the external resistor, and as we finish our trip around the loop, the potential is back where it started.

❖ *Energy and power in electric circuits*

In electric circuits we are most often interested in the rate at which energy is either delivered to or extracted from a circuit element. If the current through the element is *I* , then in a time interval *dt* an amount of charge $dQ = Idt$ passes through the element. The potential energy change for this amount of charge is $V_{ab}dQ = V_{ab}Idt$. Dividing this expression by *dt*, we obtain the *rate* at which energy is transferred either into or out of the circuit element. The time rate of energy transfer is *power*, denoted by *P*, so we write

$P = V_{ab}I$ The unit of power is Watt $1\ W = 1J/s$



➢ *Power Input to a Pure Resistance*

If the circuit element in the figure is a resistor, the potential difference is $V_{ab} = IR$. Then, the electrical power delivered to the resistor by the circuit is

$$P = V_{ab}I = I^2R = \frac{{V_{ab}}^2}{R}$$

This power is usually dissipated in the resistor as heat

Example: The power rating of a light bulb (such as a 100-W bulb) is the power it dissipates when connected across a 120-V potential difference. What is the resistance of

(a) a 100-W bulb and (b) a 60-W bulb? (c) How much current does each bulb draw in normal use?

Solution: (a) $R = \frac{V^2}{P} = \frac{(120)^2}{100} = 144\ \Omega$ (b) $R = \frac{V^2}{P} = \frac{(120)^2}{60} = 240\ \Omega$

(c) For the 100-W bulb: $I = V/R = 120/144 = 0.833\ A$

For the 60-W bulb: $I = V/R = 120/240 = 0.5\ A$

Lecture 7: Direct Current circuit

Direct Current

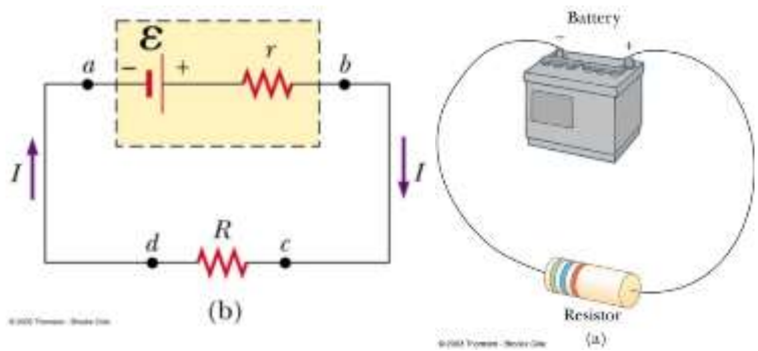
- When the current in a circuit has a constant direction, the current is called direct current
- Most of the circuits analyzed will be assumed to be in steady state, with constant magnitude and direction
- Because the potential difference between the terminals of a battery is constant, the battery produces direct current The battery is known as a source of *emf*

❖ *emf and Internal Resistance*

A real battery has some internal resistance *r* ; therefore, the terminal voltage is not equal to the emf

The terminal voltage: $\Delta V = V_b - V_a$

$$\Delta V = \varepsilon - I_r$$



For the entire circuit (*R* – load resistance):

$$\varepsilon = \Delta V + I_r$$
$$= IR + I_r$$

ε is equal to the terminal voltage when the current is zero – open-circuit voltage

$$I = \varepsilon / (R + r)$$

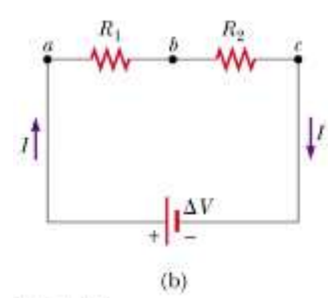
The current depends on both the resistance external to the battery and the internal resistance
When *R* >> *r*, *r* can be ignored.

Power relationship: $I\,\varepsilon = I^2 R + I^2 r$

When *R* >> *r*, most of the power delivered by the battery is transferred to the load resistor, $I^2 r$ can be ignored

❖ *Resistors in Series*

When two or more resistors are connected end-to-end, they are said to be in series.
The current is the same in all resistors because any charge that flows through one resistor flows through the other.
The sum of the potential differences across the resistors is equal to the total potential difference across the combination.



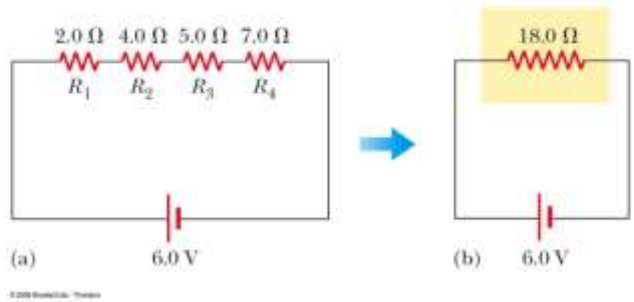
$$I_1 = I_2 = I$$

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2) = IR_{eq}$$

Where $R_{eq} = R_1 + R_2$
The equivalent resistance has the effect on the circuit as the original combination of resistors (consequence of conservation of energy)
For more resistors in series:

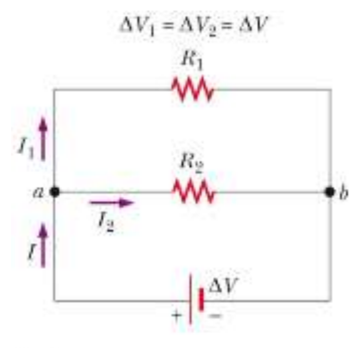
$$R_{eq} = R_1 + R_2 + R_{3+\ldots}$$

The equivalent resistance of a series combination of resistors is greater than any of the individual resistors



❖ Resistors in Parallel

The potential difference across each resistor is the same because each is connected directly across the battery terminals

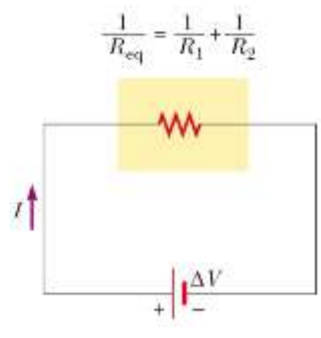


$$I_1R_1 = I_2R_2 = \Delta V$$

The current, I, that enters a point must be equal to the total current leaving that point (conservation of charge)

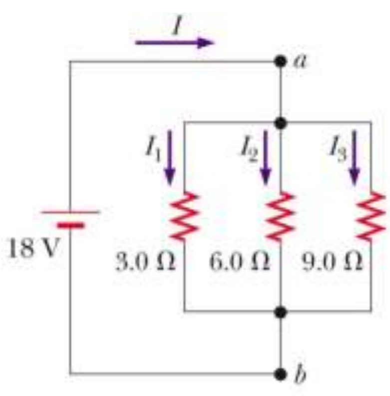
The currents are generally not the same

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- For more resistors in parallel:



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

- The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance
- The equivalent is always less than the smallest resistor in the group

➢ Problem-Solving Strategy

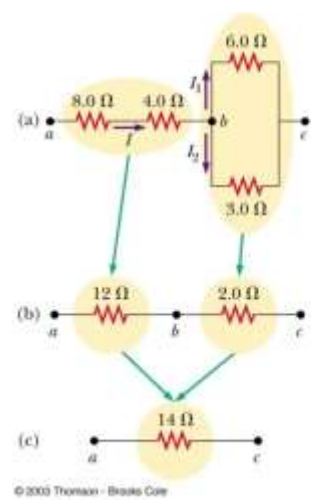
- Combine all resistors in series. They carry the same current
- The potential differences across them are not necessarily the same
- The resistors add directly to give the equivalent resistance of the combination:

$$R_{eq} = R_1 + R_2 + \ldots$$

- Combine all resistors in parallel
- The potential differences across them are the same
- The currents through them are not necessarily the same
- The equivalent resistance of a parallel combination is found through reciprocal addition:

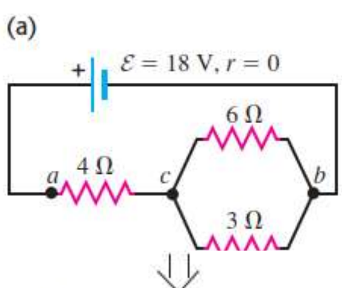
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

- A complicated circuit consisting of several resistors and batteries can often be reduced to a simple circuit with only one resistor
- Replace resistors in series or in parallel with a single resistor
- Sketch the new circuit after these changes have been made
- Continue to replace any series or parallel combinations
- Continue until one equivalent resistance is found



If the current in or the potential difference across a resistor in the complicated circuit is to be identified, start with the final circuit and gradually work back through the circuits (use formula ΔV = I R and the procedures describe above)

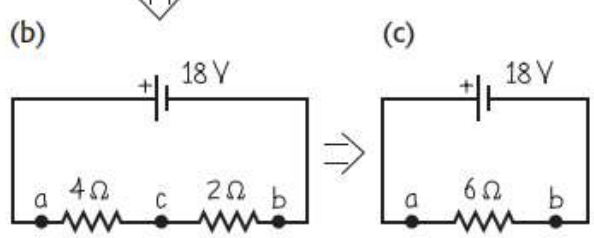
Example 1: Find the equivalent resistance of the network in the figure below and the current in each resistor. The source of ***emf*** has negligible internal resistance.



Solution:

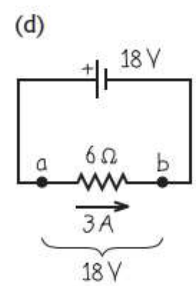
1. Reduce the parallel resistors 3 and 6;

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \qquad \therefore R = 2\,\Omega$$



2. Next get the equivalent series resistor 4 and 2;

$$R_{eq} = 4 + 2 = 6\,\Omega$$

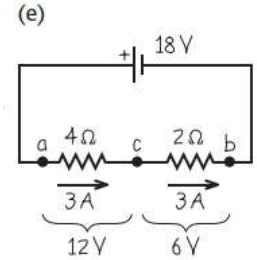


3. The current through the equivalent resistor is

$$I = \frac{V_{ab}}{R_{eq}} = \frac{18}{6} = 3\ A$$

4. The current in the 3 and 6 resistors (parallel resistors) is also 3 A. The potential difference V_{cb} across the 2Ω resistor is therefore $V_{cb} = IR = 3 \times 2 = 6\ V$. This potential difference must also be over the 6Ω.

(e)

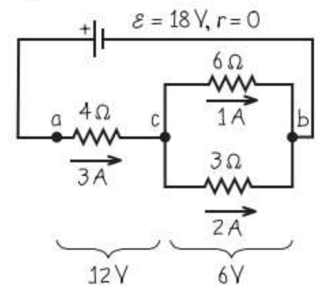


5. Thus, the current in the 6Ω and 3Ω resistors is

$$I_6 = \frac{6}{6} = 1\ A$$

$$I_3 = \frac{6}{3} = 2\ A$$

(f)

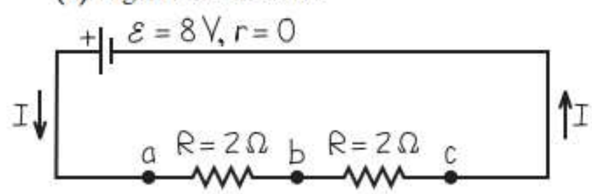


Example 2: Two identical light bulbs, each with resistance $R = 2\Omega$ are connected to a source with $\varepsilon = 8\ V$ and negligible internal resistance. Find the current through each bulb, the potential difference across each bulb, and the power delivered to each bulb and to the entire network if the bulbs are connected

(a) In series and
(b) In parallel.
(c) Suppose one of the bulbs burns out; that is, its filament breaks and current can no longer flow through it. What happens to the other bulb in the series case? In the parallel case?

Solution:

(a) Light bulbs in series



(a) The equivalent resistors for series combination is

$$R_{eq} = R_1 + R_2 = 2 + 2 = 4\Omega$$

The current is the same in both bulbs

$$I = \frac{V_{ac}}{R_{eq}} = \frac{8}{4} = 2\ A$$

Since the bulbs have the same resistance, the potential difference is the same across each bulb:

$$V_{ab} = V_{bc} = 2 \times 2 = 4\ V$$

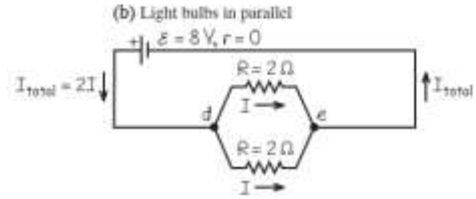
The power delivered to each bulb is

$$P = I^2R = 4 \times 2 = 8\ W$$

The total power delivered to both bulbs is

$$P_{tot} = 2P = 2 \times 8 = 16\ W$$

(b) Light bulbs in parallel



(b) For the parallel combination the potential difference V_{de} across each bulb is the same and equal to 8 V, the terminal voltage of the source. Hence the current through each light bulb is

$I = \frac{V_{de}}{R} = \frac{8}{2} = 4\ A$

and the power delivered to each bulb is

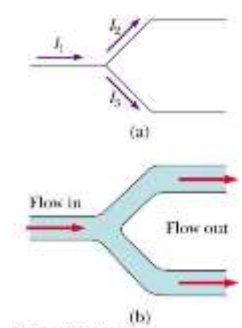
$P = I^2 R = 16 \times 2 = 32\ W$

Both the potential difference across each bulb and the current through each bulb are twice as great as in the series case. Hence the power delivered to each bulb is four times greater, and each bulb is brighter.
(c) In the series case the same current flows through both bulbs.
If one bulb burns out, there will be no current in the circuit, and neither bulb will glow.
In the parallel case the potential difference across either bulb is unchanged if a bulb burns out. The current through the functional bulb and the power delivered to it are unchanged.

Kirchhoff's Rules

- There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor
- Two rules, called Kirchhoff's Rules can be used instead:
- 1) Junction Rule
- 2) Loop Rule
- Junction Rule (A statement of Conservation of Charge):
  The sum of the currents entering any junction must equal the sum of the currents leaving that junction
- Loop Rule (A statement of Conservation of Energy):
  The sum of the potential differences across all the elements around any closed circuit loop must be zero

Kirchhoff's Junction Rule



The algebraic sum of the currents into any junction is zero. That is, $\sum I = 0$

or $I_1 = I_2 + I_3$

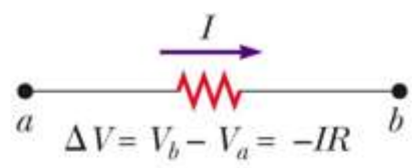
- Assign symbols and directions to the currents in all branches of the circuit.
- If a direction is chosen incorrectly, the resulting answer will be negative, but the magnitude will be correct

Kirchhoff's Loop Rule

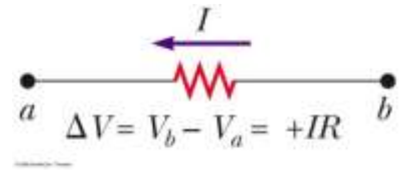
The algebraic sum of the potential differences in any loop, including those associated with ***emf***s and those of resistive elements must equal zero. That is $\sum V = 0$

Sign Conventions for the Loop Rule

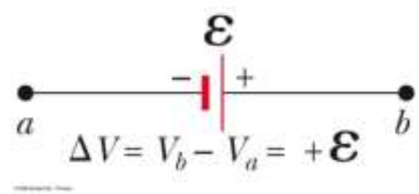
- When applying the loop rule, choose a direction for traveling the loop and record voltage drops and rises as they occur.
- Starting at any point in the circuit, we imagine traveling around a loop, adding ***emf***s and *IR* terms as we come to them.
- If we travel through a resistor in the same direction as the current, the potential across the resistor is decreasing ($-IR$)



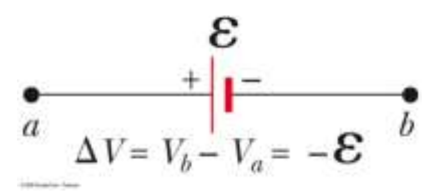
- If we travel through a resistor is in the direction opposite to the current, the potential across the resistor is increasing ($+IR$)



- If a source of ***emf*** is in the direction of the ***emf*** ($from - to +$), the change in the electric potential is $+\varepsilon$



- If a source of ***emf*** is in the direction opposite to the ***emf*** ($from + to -$), the change in the electric potential is $-\varepsilon$



Problem-Solving Strategy

- Draw the circuit diagram and assign labels and symbols to all known and unknown quantities
- Assign directions to the currents
- Apply the junction rule to any junction in the circuit
- Apply the loop rule to as many loops as are needed to solve for the unknowns
- Solve the equations simultaneously for the unknown quantities
- Check your answers

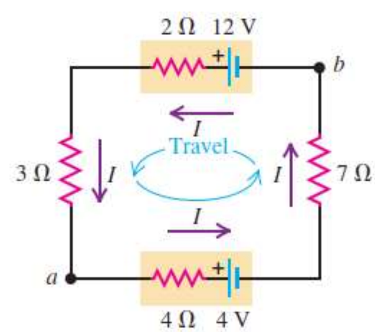
Example 1: The circuit shown in Figure contains two batteries, each with an ***emf*** and an internal resistance, and two resistors.

Find

(a) The current in the circuit,

(b) The potential difference and The power output of the ***emf*** of each battery

Solution:



(a) Starting at a and traveling counterclockwise with the current, we add potential increases and decreases and equate the sum to zero. Then

$$-4I - 4 - 7I + 12 - 2I - 3I = 0$$

$$-16I = 8$$

$$I = 0.5\ A$$

Positive result for I shows that our assumed current direction is correct

(b) To find the potential difference V_{ab} the potential at a with respect to b, we start at b and add potential changes as we go toward a. There are two paths from b to a; Taking the upper path from point b to a, we find

$$V_{ab} = 12 - (0.5 \times 2) - (0.5 \times 3) = 9.5\ V$$

Here the IR terms are negative because our path goes in the direction of the current, with potential decreases through the resistors.

If we take the lower path from b to a, we find

$$V_{ab} = (0.5 \times 7) + 4 + (0.5 \times 4) = 9.5\ V$$

The results for V_{ab} are the same for both paths, as they must be in order for the total potential change around the loop to be zero.

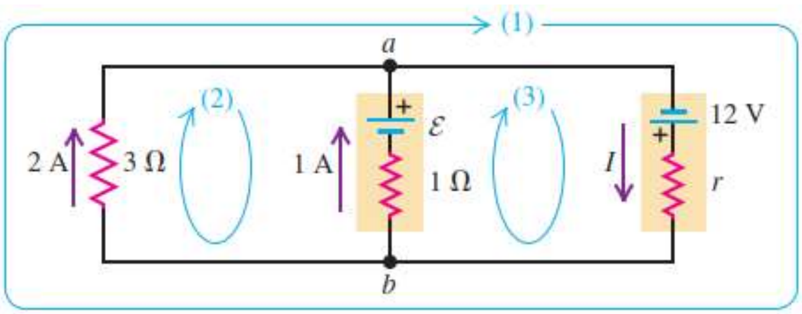
(c) The power outputs of the ***emf*** of the 12-V and 4-V batteries are

$$P_{12} = \varepsilon I = 12 \times 0.5 = 6\ V$$

$$P_4 = \varepsilon I = -4 \times 0.5 = -2\ V$$

The negative sign in ε for the 4-V battery appears because the current actually runs from the higher-potential side of the battery to the lower-potential side. The negative value of P means that we are storing energy in that battery; the 12-V battery is recharging it (if it is in fact rechargeable; otherwise, we're destroying it).

Example 2: In the circuit shown in the figure below, a 12-V power supply with unknown internal resistance r is connected to a run-down rechargeable battery with unknown ***emf*** ε and internal resistance 1Ω and to an indicator light bulb of resistance 3Ω carrying a current of 2 A. The current through the run-down battery is 1 A in the direction shown.
Find r, ***emf*** , and the current I through the power supply.



Solution: We apply the junction rule to point a

$$-I + 1 + 2 = 0$$

$$\therefore I = 3A$$

To determine r, we apply the loop rule to the large, outer loop (1):

$$12 - 3r - 2 \times 3 = 0$$

$$\therefore r = 2\ \Omega$$

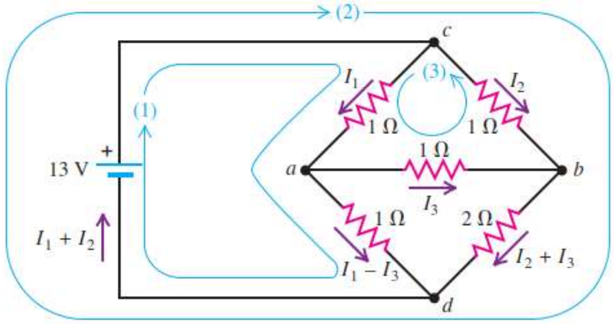
To determine ε we apply the loop rule to the left-hand loop (2):

$$-\varepsilon + (1A \times 1\Omega) - (2A \times 3\Omega) = 0$$

$$\therefore \varepsilon = -5V$$

The negative value for ε shows that the actual polarity of this ***emf*** is opposite to that shown in the figure, the battery is being recharged

Example 3: The figure shows a "bridge" circuit. Find the current in each resistor and the equivalent resistance of the network of five resistors



Solution: We apply the loop rule to the three loops shown:

$$13\text{ V} - I_1(1\ \Omega) - (I_1 - I_3)(1\ \Omega) = 0 \qquad (1)$$

$$-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + 13\text{ V} = 0 \qquad (2)$$

$$-I_1(1\ \Omega) - I_3(1\ \Omega) + I_2(1\ \Omega) = 0 \qquad (3)$$

Solve these simultaneous equations for the currents;

From eqn. (3) $I_2 = I_1 + I_3$ and then substitute this expression into Eq. (2) to eliminate I_2 . We then have

$$-(I_1 + I_3) - 2(I_1 + I_3) - 2I_3 + 13 = 0$$

$$3I_1 + 5I_3 = 13 \qquad (4)$$

From eqn. (1) $2I_1 - I_3 = 13$ (1') $\times 5$

$$13I_1 = 78$$

$$\therefore I_1 = 6A$$

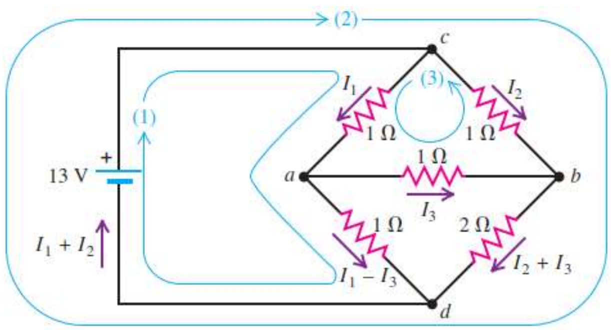
substitute this result into Eqn. (1') $I_3 = -1\text{A}$

And from Eqn. (3) we find $I_2 = 5A$

The negative value of I_3 shows that its direction is opposite to the direction we assumed. The total current through the network is $I_1 + I_2 = 11A$. And the potential drop across it is equal to the battery emf, 13 V. The equivalent resistance of the network is therefore

$$R_{eq} = \frac{13}{11} = 1.2\ \Omega$$

Example 4: use the results from example 3 to find the potential difference V_{ab}



Solution: $V_{ab} = V_a - V_b$ is the potential at point a with respect to point b. To find it, we start at point b and follow a path to point a, adding potential rises and drops as we go. We can follow any of several paths from b to a; the result must be the same for all such paths, which gives us a way to check our result.

The simplest path is through the center 1Ω resistor.

In Example 3 we found $I_3 = -1\ A$, showing that the actual current direction through this resistor is from right to left.

Thus, as we go from b to a, there is a drop of potential with magnitude

$V_{ab} = I_3 R = -1 \times 1 = -1\, V$,

The potential at a is 1 V less than at point b.

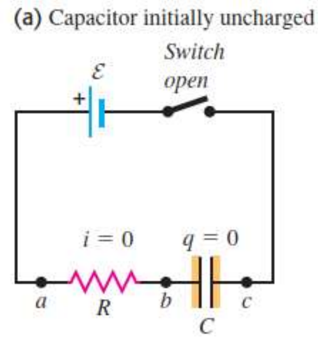
Lecture 8: RC circuits

RC Circuits

A circuit that has a resistor and a capacitor in series is called an R-C circuit.

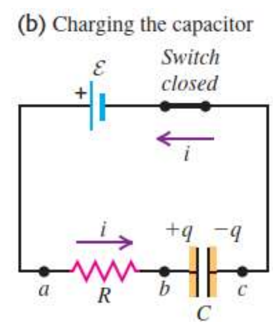
- Capital letters: *V, Q, I* (constant)
- Lowercase letters: *v, i, q* (vary with time)

❖ Charging a Capacitor:



(a) Because the capacitor is initially uncharged, the potential difference $v_{bc} = 0$ at $t = 0$. at this time, from Kirchhoff's loop law, the voltage v_{ab} across the resistor R is equal to the battery ***emf*** $\boldsymbol{\varepsilon}$. The current through the resistor is given by Ohm's law

$$I_0 = v_{ab}/R = \varepsilon/R$$



(b) As the capacitor charges, its voltage v_{bc} increases and v_{ab} decreases. At an intermediate time, *t,* let q represent the charge on the capacitor, then, the instantaneous potential differences

$$v_{ab} = iR$$
$$v_{bc} = q/C$$

Using these in Kirchhoff's loop rule, we find that The potential drops by an amount iR as we travel from a to b and by q/C as we travel from b to c.

$$\therefore \varepsilon - iR - \frac{q}{C} = 0 \qquad \therefore i = \frac{\varepsilon}{R} - \frac{q}{RC}$$

As the charge q increases, the term q/RC becomes larger and the capacitor charge approaches its final value Q_f. The current decreases and eventually becomes zero. When $i = 0$

$$\frac{\varepsilon}{R} = \frac{Q_f}{RC}\text{, so } Q_f = C\varepsilon$$

Note that the final charge does not depend on R.

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\varepsilon)$$

$$\frac{dq}{q - C\varepsilon} = -\frac{1}{RC}$$

and then integrate both sides. We change the integration variables to q' and t' so that we can use q and t for the upper limits. The lower limits are $q' = 0$ and $t' = 0$

$$\int_0^q \frac{dq'}{q' - C\varepsilon} = -\int_0^t \frac{dt'}{RC}$$

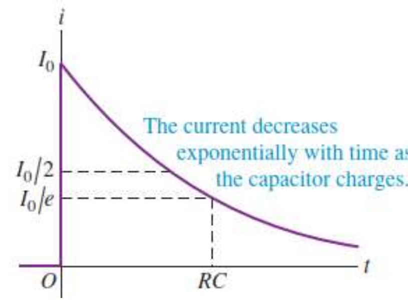
$$ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

Taking the exponential of both sides and solving for q

$$\frac{q - C\varepsilon}{-C\varepsilon} = e^{-\frac{t}{RC}}$$

$$\therefore q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$$

(a) Graph of current versus time for a charging capacitor



The instantaneous current i is just the time derivative of q

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC}$$

Time Constant

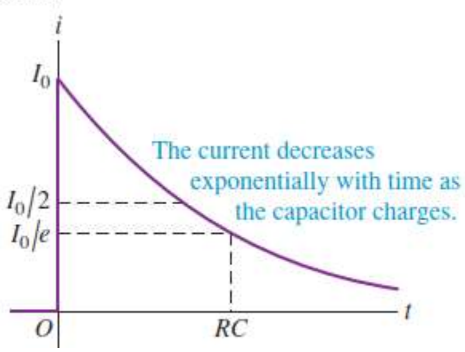
After a time equal to RC, the current in the R-C circuit has decreased to $1/e$ (about 0.368) of its initial value. At this time, the capacitor charge has reached $(1 - 1/e) = 0.632$ of its final value Q_f. The product RC is therefore a measure of how quickly the capacitor charges. We call RC the ***time constant*** or the ***relaxation time*** of the circuit, denoted by τ;

$$\tau = RC$$

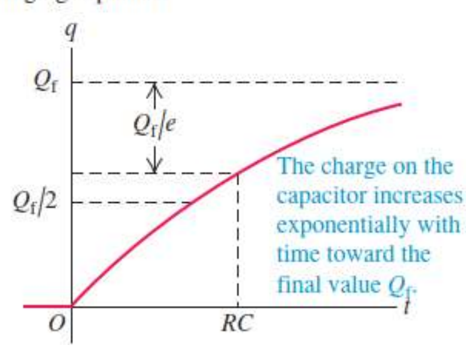
$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC}$$

$$\therefore q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$$

(a) Graph of current versus time for a charging capacitor



(b) Graph of capacitor charge versus time for a charging capacitor

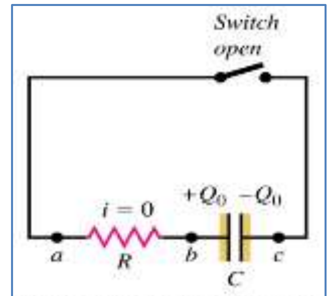


المادة: الفيزياء 2 المرحلة الاولى

د. وسام عبدالله لطيف

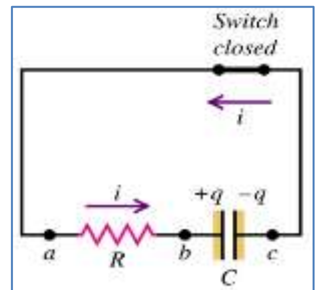
Discharging a Capacitor:

Now suppose that after the capacitor in (a) has acquired a charge Q_0 we remove the battery from our R-C circuit and connect points *a* and *c* to an open switch.



(a) Capacitor initially charged

We then close the switch and at the same instant reset the time to $t = 0$ at that time, the capacitor then discharges through the resistor, and its charge eventually decreases to zero.

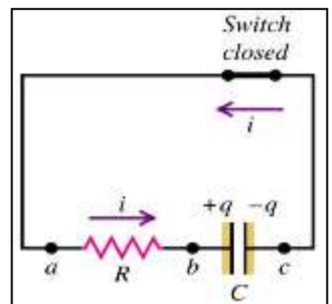


(b) Discharging the capacitor

Again let *i* and *q* represent the time-varying current and charge at some instant after the connection is made. We make the same choice of the positive direction for current. Then Kirchhoff's loop rule with $\varepsilon = 0$ now gives:

$$0 = v_{ab} + v_{bc}$$

$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C} \qquad \therefore i = \frac{dq}{dt} = -\frac{q}{RC}$$



(b) Discharging the capacitor

The current *i* is now negative; this is because positive charge q is leaving the left hand capacitor plate, so the current is in the direction opposite to the charge direction

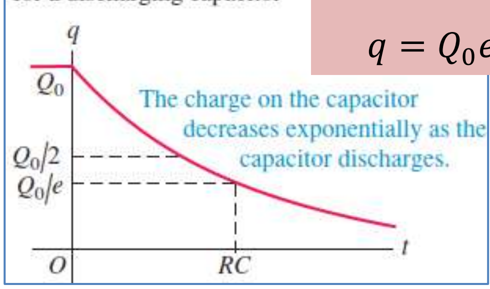
At time $t = 0$, $q = Q_0$, the initial current is $I_0 = -Q_0/RC$.
To find *q* as a function of time, again change the limits to *q'* and *t'* and integrate

$$\int_{Q_0}^{q} \frac{dq}{q'} = -\frac{1}{RC}\int_0^t dt' \Rightarrow ln\frac{q}{Q_0} = -\frac{t}{RC} \Rightarrow q = Q_0 e^{-\frac{t}{RC}}$$

The instantaneous current *i* is $$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$

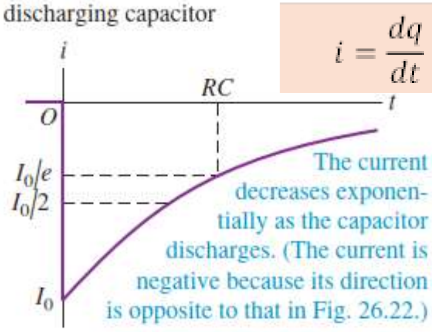
(b) Graph of capacitor charge versus time for a discharging capacitor

$$q = Q_0 e^{-\frac{t}{RC}}$$



(a) Graph of current versus time for a discharging capacitor

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$



During charging:

The instantaneous rate at which battery delivers energy to circuit

$$\varepsilon i = i^2 R + \frac{iq}{C}$$

i^2R = power dissipated in R

iq/C = power stored in C

Total energy supplied by battery: εQ_f

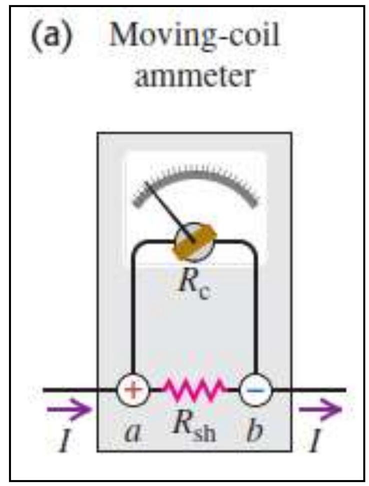
Total energy stored in capacitor: $Q_f\,\varepsilon/2$

Electrical Measuring Instruments

Ammeter: device that measures current, $(R = 0)$

- It can be adapted to measure currents larger than its full scale range by connecting R_{sh} (shunt resistor) in parallel (some I bypasses meter coil).

$$I_a = I_{sh} + I_{fs}$$



I_{fs} = current through coil

I_{sh} = current through R_{sh}

I_a = current measured by ammeter

The potential difference V_{ab} is the same for both paths, so

$$I_{fs}R_c = I_{sh}R_{sh}$$

$$V_{ab} = I_{fs}R_c = (I_a - I_{fs})R_{sh}$$

Example: What shunt resistance is required to make the $1mA$, 20Ω meter described above into an ammeter with a range of 0 to 50 mA?

Solution: $I_{fs} = 1\times10^{-3}A$, $I_a = 50\times10^{-3}A$, $R_c = 20\Omega$

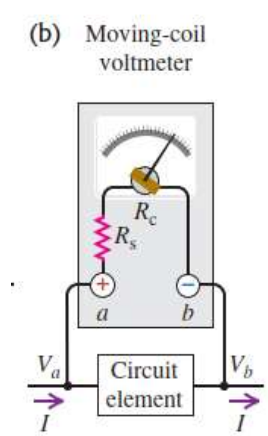
$$I_{fs}R_c = (I_a - I_{fs})R_{sh}$$

$$R_{sh} = \frac{I_{fs}R_c}{(I_a - I_{fs})} = \frac{1\times10^{-3}\times20}{(50-1)\times10^{-3}} = 0.408\Omega$$

Voltmeter: device that measures voltage, $(R = \infty)$

- It can be adapted to measure voltages larger than its full scale range by connecting R_s in series with the coil.

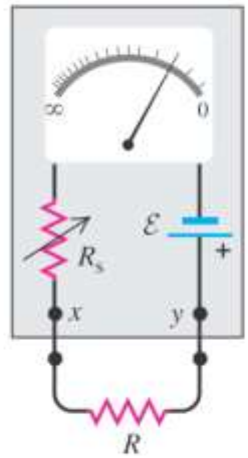
$$V_v = V_{ab} = I_{fs}(R_c + R_s)$$



Example: What series resistance is required to make the $1.mA$, 20Ω meter described above into a voltmeter with a range of 0 to 10 V?

Solution:

$$R_s = \frac{V_v}{I_{fs}} - R_c = \frac{10}{1\times10^{-3}} - 20 = 9980\Omega$$

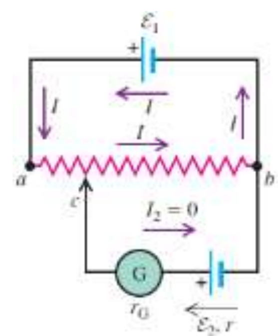


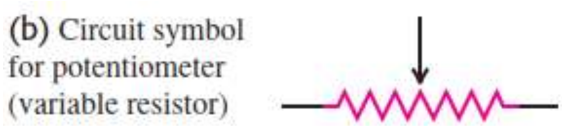
Ohmmeter: device that measures resistance.

- The series resistance R_s is adjusted so that when the terminals x-y are short-circuited ($R = 0$), the meter deflects full scale (zero). When nothing is connected between x-y (open circuit, $R = \infty$) there is no current (no deflection). For intermediate R values, meter scale is calibrated to read R.

Potentiometer: device that measures ***emf*** of a source without drawing any current from it.

- R_{ab} connected to terminals of known emf (ε_1). A sliding contact (c) is connected through galvanometer (G) to unknown source (ε_2). As contact (c) is moved along R_{ab}, R_{cb} varies proportional to wire length (c-b). To find ε_2 (c) is moved until G shows no deflection ($I_G = 0$):

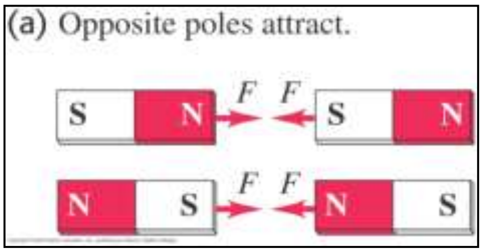


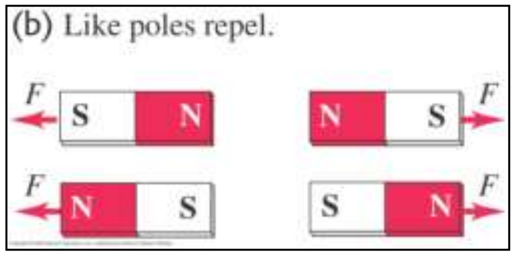


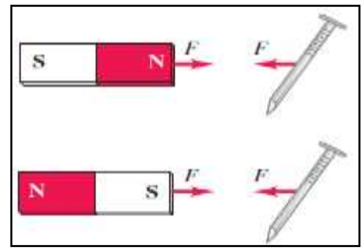
Lecture 9. Magnetic field and magnetic forces

Magnetism:

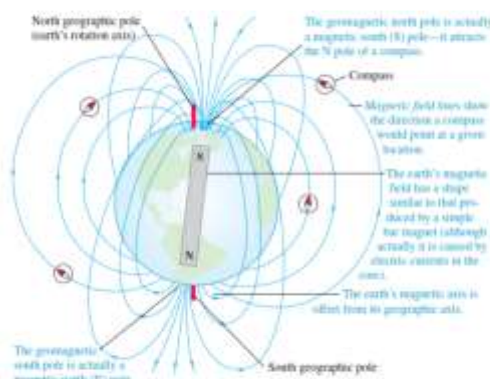
- Magnets exert forces on each other just like charges. You can draw magnetic field lines just like you drew electric field lines.
- Magnetic north and south pole's behavior is similar to electric charges. For magnets, like poles repel and opposite poles attract.
- A permanent magnet will attract a metal like iron with either the north or south pole.







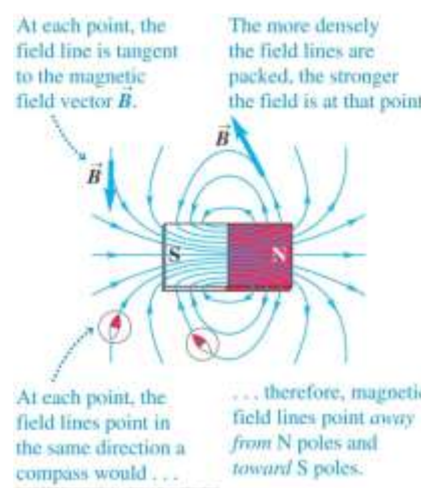
The earth's magnetic field



- Magnetic declination / magnetic variation: the Earth's magnetic axis is not parallel to its geographic axis (axis of rotation) a compass reading deviates from geographic north.
- Magnetic inclination: the magnetic field is not horizontal at most of earth's surface, its angle up or down. The magnetic field is vertical at magnetic poles.

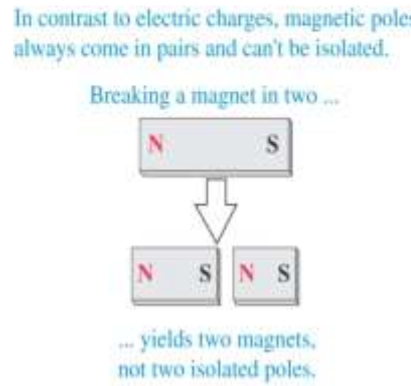
MAGNETIC FIELD LINES

We can represent any magnetic field by *magnetic field lines*, just as we did for the earth's magnetic field. The idea is the same as for the electric field lines. We draw the lines so that the line through any point is tangent to the magnetic field vector at that point as shown in the figure. Just as with electric field lines, we draw only a few representative lines; otherwise, the lines would fill up all of space. Where adjacent field lines are close together, the field magnitude is large; where these field lines are far apart, the field magnitude is small. Also, because the direction of at each point is unique, field lines never intersect.



Magnetic Poles versus Electric Charge

- We observed monopoles in electricity. A (+) or (-) alone was stable, and field lines could be drawn around it.
- Magnets cannot exist as monopoles. If you break a bar magnet between N and S poles, you get two smaller magnets, each with its own N and S pole.



Electric field	Magnetic field
1) A distribution of electric charge at rest creates an electric field $\boldsymbol{E}$ in the surrounding space.	1) A moving charge or current creates a magnetic field in the surrounding space (in addition to $\boldsymbol{E}$).
2) The electric field exerts a force $\boldsymbol{F_E} = q\,\boldsymbol{E}$ on any other charges in presence of that field.	2) The magnetic field exerts a force $\boldsymbol{F_m} = q\boldsymbol{v} \times \boldsymbol{B}$ on any other moving charge or current present in that field.

- The magnetic field is a vector field vector quantity associated with each point in space.

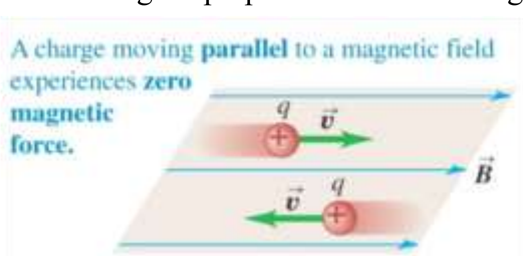
$$F_m = |q|v_{\perp}B = |q|vBsin\varphi$$

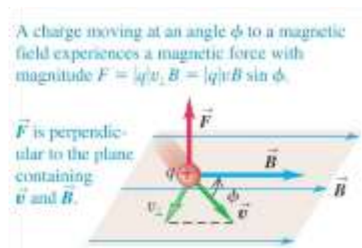
$$\boldsymbol{F_m} = |q|\boldsymbol{v} \times \boldsymbol{B}$$

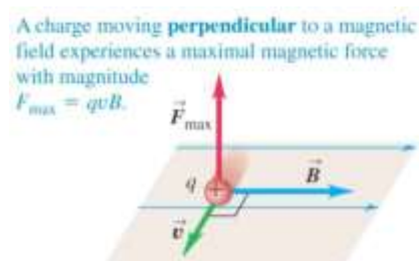
- $\boldsymbol{F_m}$ is always perpendicular to $\boldsymbol{B}$ and $\boldsymbol{v}$.

Interaction of magnetic force and charge

The moving charge interacts with the fixed magnet. The force between them is at a maximum when the velocity of the charge is perpendicular to the magnetic field.

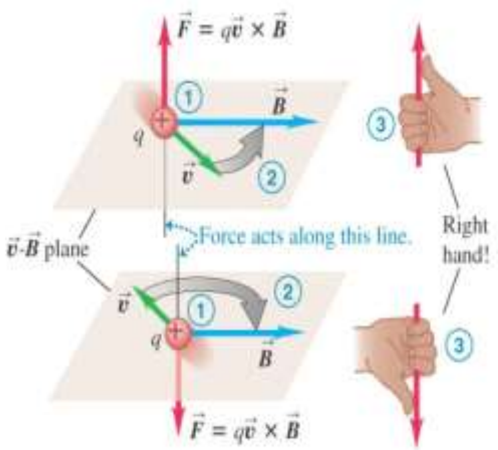




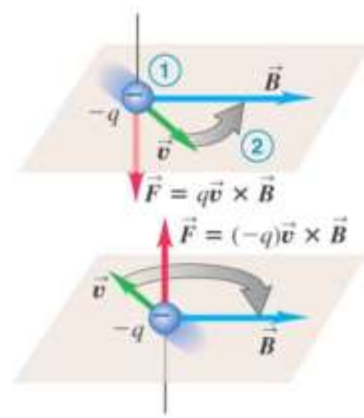


Right Hand Rule

Positive charge moving in magnetic field
→direction of force follows right hand rule

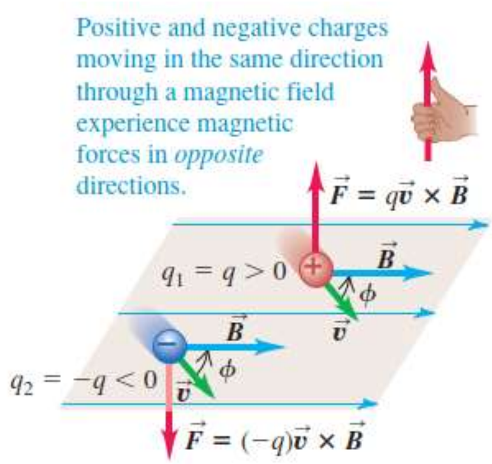


Negative charge → F direction contrary to right hand rule.



Units: $1\,Tesla = 1\,N\,s\,/\,C\,m = 1\,N/A\,m$

$1\,Gauss = 10^{-4}\,T$



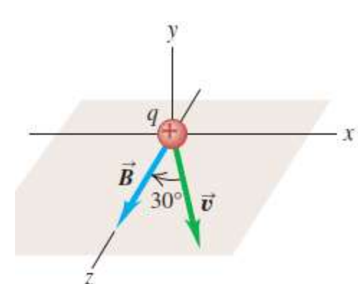
Two charges of equal magnitude but opposite signs moving in the same direction in the same field will experience force in opposing directions.

If charged particle moves in region where both, ***E*** and ***B*** are present:

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

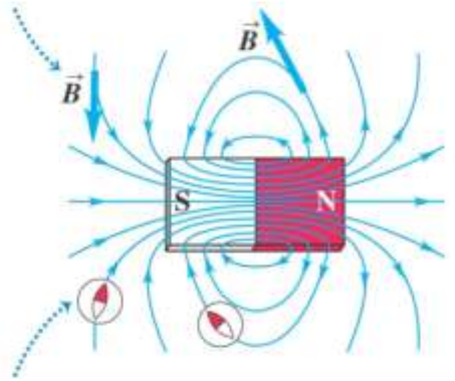
Example: A beam of protons ($q = 1.6 \times 10^{-19} C$) moves at $3 \times 10^5 m/s$ through a uniform 2-T magnetic field directed along the positive z-axis. The velocity of each proton lies in the xz-plane and is directed at 30^0 to the positive z-axis. Find the force on a proton.

Solution: The charge is positive, so the force is in the same direction as the vector product $\boldsymbol{v} \times \boldsymbol{B}$. From the right-hand rule, this direction is along the negative y-axis.



$$F = qvBsin\varphi$$
$$F = 1.6 \times 10^{-19} \times 3 \times 10^5 \times 2 \times \sin 30$$
$$= 4.8 \times 10^{-14} N$$

Magnetic Field Lines and Magnetic Flux

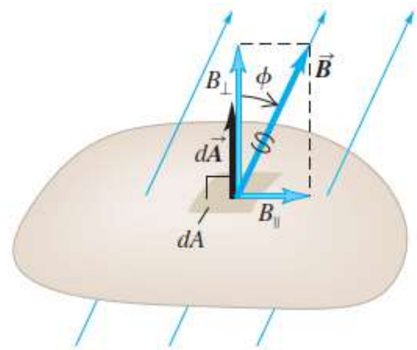


- Magnetic field lines may be traced from N toward S (analogous to the electric field lines).
- At each point they are tangent to magnetic field vector.
- The more densely packed the field lines, the stronger the field at a point.
- Field lines never intersect.
- The field lines point in the same direction as a compass (from N toward S).
- Magnetic field lines are not "lines of force".
- Magnetic field lines have no ends, so they continue through the interior of the magnet.

Magnetic Flux and Gauss's Law for Magnetism

We define the **magnetic flux** ϕ_B through a surface just as we defined electric flux in connection with Gauss's law. We can divide any surface into elements of area dA. For each element we determine $B_\perp$ the component of ***B*** normal to the surface at the position of that element, as shown in the figure below. From the figure $B_\perp = B\cos\varphi$, where φ is the angle between the direction of ***B*** and a line perpendicular to the surface. We define the magnetic flux $d\phi_B$ through this area as

$$d\Phi_B = B_\perp\, dA = B\cos\phi\, dA = \vec{B}\cdot d\vec{A}$$



The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\Phi_B = \int B_\perp\, dA = \int B\cos\phi\, dA = \int \vec{B}\cdot d\vec{A}$$

- Magnetic flux is a scalar quantity.
- If ***B*** is uniform, then

$$\phi_B = B_\perp A = BA\cos\varphi$$

- If ***B*** happens to be perpendicular to the surface, then $\varphi = 0\ \cos\varphi = 1$ and

  $\phi_B = \mathrm{BA}$
- The SI unit of magnetic flux is equal to the unit of magnetic field (1 T) times the unit of area This unit is called the **weber** $1\,Wb = 1T.m^2 = 1N.m/A$

The total magnetic flux through a closed surface is always zero. This is because there is no isolated magnetic charge ("monopole") that can be enclosed by the Gaussian surface.

$$\oint B\cdot dA = 0$$

Example: A circular area with a radius of 6.5 cm lies in the xy-plane. What is the magnitude of the magnetic flux through this circle due to a uniform magnetic field $B = 0.23\,T$.

(a) In the +z-direction;

(b) At an angle of 53.1^0 from the +z-direction;

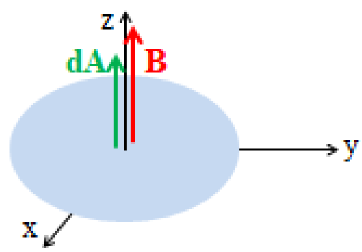
(c) In the +y-direction ?

Solution: Circular area in the *xy*-plane, so $A = \pi r^2$

$$A = \pi (0.065\,m)^2 = 0.01327\,m^2$$

and $d\boldsymbol{A}$ is in the z- direction

(a) $\boldsymbol{B} = (0.23)\boldsymbol{k}$; so $\boldsymbol{B}$ and $d\boldsymbol{A}$ are parallel $(\varphi = 0)$.

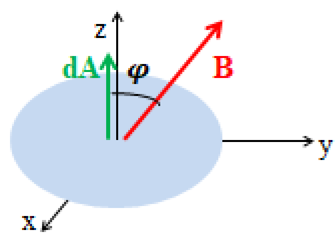


$$\Phi_B = \int B \cdot dA = B\cos\varphi \int dA = BA$$

$$= 0.23 \times 0.01327$$

$$= 3.05 \times 10^{-3}\ Wb$$

(b) $\varphi = 53.1^0$



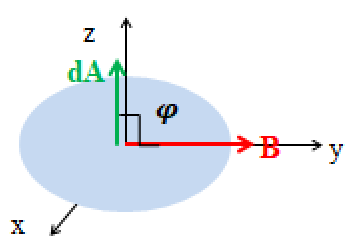
$$\Phi_B = \int B \cdot dA = B\cos\varphi \int dA$$

$$\Phi_B = B\cos\varphi A$$

$$\Phi_B = (0.23)\cos(53.1)(0.01372)$$

(c) Since B and dA are perpendicular$(\varphi = 90^0)$

$$\Phi_B = \int B \cdot dA = B\cos(90) \int dA = 0$$

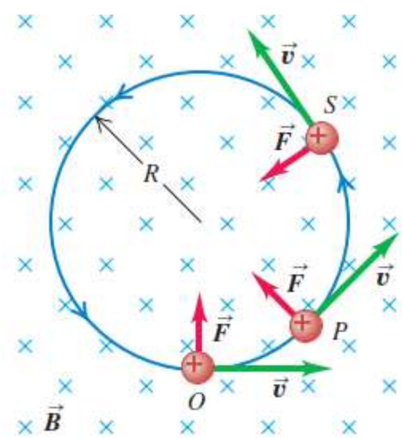


Motion of charged particles in a magnetic field

- When a charged particle moves in a magnetic field, it is acted on by the magnetic force $(\boldsymbol{F_m} = q\boldsymbol{v} \times \boldsymbol{B})$ and the motion is determined by Newton's laws.
- The force is perpendicular to the velocity, so the charged particle experiences an acceleration that is perpendicular to the velocity.
- The magnitude of the velocity does not change, but the direction of the velocity does producing circular motion.
- The magnetic force does no work on the particle.
- The magnetic force produces circular motion with the centripetal acceleration being given by

$$a = \frac{v^2}{R}$$

where R is the radius of the orbit



- Using Newton's second law we have

$$F_m = qvB = m\frac{v^2}{R}$$

- The radius of the orbit is then given by

$$R = \frac{mv}{qB}$$

$+q$ ➡ Counter-clockwise rotation.
$-q$ ➡ Clockwise rotation.

- The angular speed ω is given by

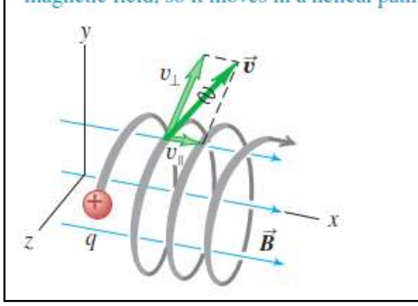
$$\omega = \frac{v}{R} = \frac{qB}{m}$$

- The frequency

$$f = \frac{\omega}{2\pi}$$

- What is the motion like if the velocity is not perpendicular to B?
- We break the velocity into components along the magnetic field and perpendicular to the magnetic field.
- The component of the velocity perpendicular to the magnetic field will still produce circular motion.
- The component of the velocity parallel to the field produces no force and this motion is unaffected
- The combination of these two motions results in a helical type motion

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.



Example 1: A magnetron in a microwave oven emits electromagnetic waves with frequency $f = 2450\ MHz$. What magnetic field strength is required for electrons to move in circular paths with this frequency?
$m_e = 9.11 \times 10^{-31} kg$

Solution: The angular speed that corresponds to the frequency

$$\omega = 2\pi f = 2\pi \times 2450 \times 10^6 = 1.54 \times 10^{10} s^{-1}$$

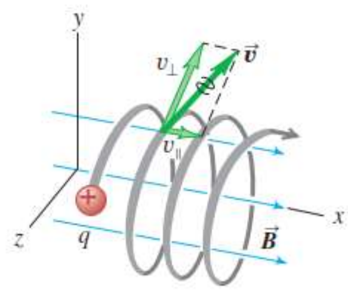
$$\omega = \frac{qB}{m}$$

$$\therefore B = \frac{m\omega}{q} = \frac{9.11 \times 10^{-31} \times 1.54 \times 10^{10}}{1.6 \times 10^{-19}} = 0.0877\ T$$

Example 2: In a situation like that shown in the figure, the charged particle is a proton $\left(q = 1.6 \times 10^{-19} C,\ m = 1.67 \times 10^{-27} kg\right)$ and the uniform, $0.5T$ magnetic field is directed along the x-axis. At $t = 0$ the proton has velocity components $v_x = 1.5 \times 10^5 m/s$, and $v_y = 0$, $v_z = 2 \times 10^5 m/s$. Only the magnetic force acts on the proton.

(a) At $t = 0$, find the force on the proton and its acceleration.

(b) Find the radius of the resulting helical path, the angular speed of the proton, and the *pitch* of the helix (the distance traveled along the helix axis per revolution).



Solution: (a) $\boldsymbol{B} = B\boldsymbol{i}$ and $\boldsymbol{v} = \boldsymbol{v_x i} + \boldsymbol{v_z k}$

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B} = \boldsymbol{q}(\boldsymbol{v_x i} + \boldsymbol{v_z k}) \times B\boldsymbol{i}$$

Recall $\boldsymbol{i} \times \boldsymbol{i} = 0$ and $\boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}$

$$\therefore F = qv_z B\boldsymbol{j}$$
$$= (1.6 \times 10^{-19})(2 \times 10^5)(0.5)$$
$$= (1.6 \times 10^{-14})\boldsymbol{j}$$

From Newton 2nd law, the resulting acceleration is

$$\boldsymbol{a} = \frac{\boldsymbol{F}}{m} = \frac{1.6 \times 10^{-14}}{1.67 \times 10^{-27}} = (9.58 \times 10^{12})\boldsymbol{j}$$

(b) Since $v_y = 0$, the component of velocity perpendicular to $\boldsymbol{B}$ *is* v_z, then the radius R is

$$R = \frac{mv_z}{|q|B} = \frac{(1.67 \times 10^{-27})(2 \times 10^5)}{(1.6 \times 10^{-19})(0.5)}$$
$$= 4.18 \times 10^{-3} m = 4.18\ mm$$

- ❑ The angular speed is

$$\omega = \frac{|q|B}{m} = \frac{(1.6 \times 10^{-19})(0.5)}{1.67 \times 10^{-27}} = 4.79 \times 10^7 rad/s$$

- ❑ The period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.79 \times 10^7} = 1.31 \times 10^{-7} s$$

- ❑ The pitch is the distance traveled along the x-axis in this time,

$$v_x T = (1.5 \times 10^5)(1.31 \times 10^{-7}) = 19.7\ mm$$

Lecture 10. Applications of Charged Particles Motion

➢ *Velocity Selector*

An interesting device can be built that uses both magnetic and electric fields that are perpendicular to each other.

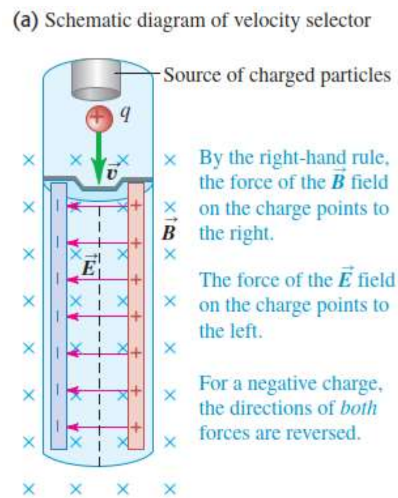
A charged particle entering this device with a velocity ***v*** will experience both an electric force $F_E = qE$ and a magnetic force $F_B = qvB$

If the particle is positively charged then the magnetic force on the particle will be to the right and the electric force will be to the left. If the velocity of the charged particle is just right then the net force on the charged particle will be zero $\sum F = F_B - F_E = 0$

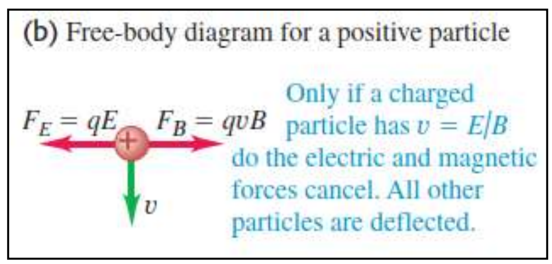
$$\sum F = F_B - F_E = 0$$

$$\therefore qvB = qE$$

$$\therefore v = \frac{E}{B}$$

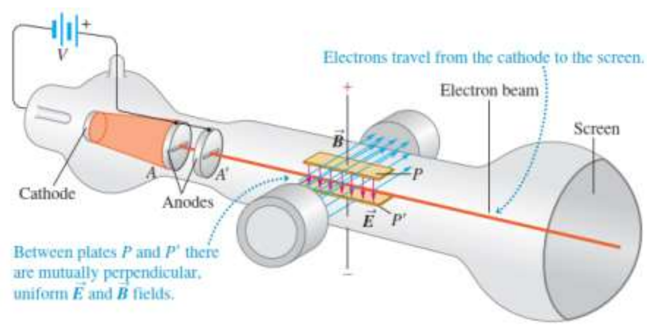


- Only particles with speeds equal to can pass through without being deflected by the fields.
- By adjusting ***E*** and ***B*** appropriately, we can select particles having a particular speed for use in other experiments. Because q cancels out , a velocity selector for positively charged particles also works for electrons or other negatively charged particles.



➢ Thomson's e/m Experiment

In a highly evacuated glass container, electrons from the hot cathode are accelerated and formed into a beam by a potential difference V between the two anodes A and A'. The speed v of the electrons is determined by the accelerating potential V. The gained kinetic energy $\frac{1}{2}mv^2$ equals the lost electric potential energy eV where e is the magnitude of the electron charge:



$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

The electrons pass between the plates P and P' and strike the screen at the end of the tube, which is coated with a material that fluoresces (glows) at the point of impact. The electrons pass straight through the plates when Eq. $v = E/B$ is satisfied. Therefore,

$$\frac{E}{B} = \sqrt{\frac{2eV}{m}} \Rightarrow \frac{e}{m} = \frac{E^2}{2VB^2}$$

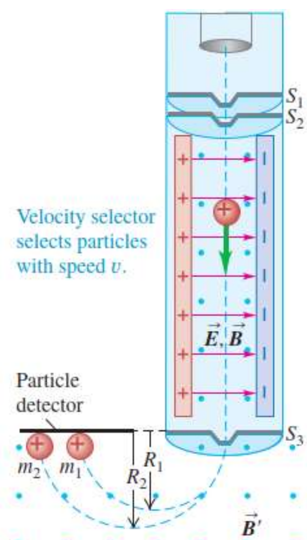
All the quantities on the right side can be measured, so the ratio e/m of charge to mass can be determined. The most precise value of available as of this writing is

$$e/m = 1.758820150(44) \times 10^{11}\ \text{C/kg}$$

$$m = 9.10938215(45) \times 10^{-31}\ \text{kg}$$

- ***Mass Spectrometer***

Using the same concept as Thompson, Positive ions from a source pass through the slits S_1 and S_2 forming a narrow beam. Then the ions pass through a velocity selector with crossed E and B fields. Finally, the ions pass into a region with a magnetic field perpendicular to the figure, where they move in circular arcs with radius R determined by $R = mv/qB'$ the values of R can be measured. We assume that each ion has lost one electron, so the net charge of each ion is just $+e$.
With everything known in this equation except m we can compute the mass of the ion. $v = E/B$. After this, in the region of B' particles with $m_2 > m_1$ travel with radius $(R_2 > R_1)$.



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

Example: You set out to reproduce Thomson's e/m experiment with $e/m = 1.758820150(44) \times 10^{11}\ \text{C/kg}$ and an accelerating potential of $150\ V$ and a deflecting electric field of magnitude $6 \times 10^6\ N/C$

(a) At what fraction of the speed of light do the electrons move?
(b) What magnetic-field magnitude will yield zero beam deflection?

Solution: (a) The electron speed is given by

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{2 \times (1.76 \times 10^{11}) \times 150} = 7.27 \times 10^6\ m/s$$

$$\frac{v}{c} = \frac{7.27 \times 10^6}{3 \times 10^8} = 0.027$$

Therefore the electron moves with 0.027 of speed of light.

$$B = \frac{E}{v} = \frac{6 \times 10^6}{7.27 \times 10^6} = 0.83\ T$$

Magnetic Force on a Current-Carrying Conductor

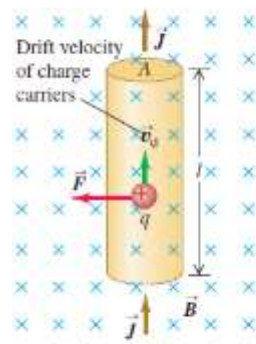
- The average magnetic force on a single moving charge is

$$\boldsymbol{F_m} = q\boldsymbol{v_d} \times \boldsymbol{B}$$

- Since v and $\boldsymbol{B}$ are perpendicular, the magnitude of the force is

$$F_m = qvB$$

- The total force on all the moving charges in a length l of conductor with cross-sectional area A



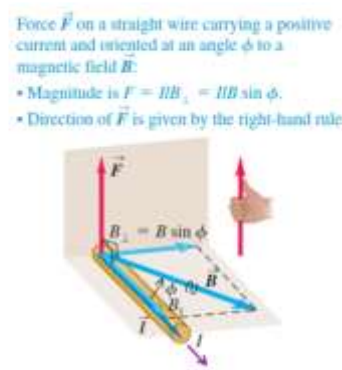
$$F = (nlA)(qv_dB)$$

- Where Al is the volume of the conductor and n is the number of charges per unit volume.

The current density is $J = \frac{I}{A} = nqv_d$ ⟶ $I = Anqv_d$

$$\therefore F = IlB$$

- If the field $\boldsymbol{B}$ ***is*** not perpendicular to the wire but makes an angle φ with it. Then, only the component of $\boldsymbol{B}$ perpendicular to the wire (and to the drift velocities of the charges) exerts a force; this component is $\boldsymbol{B}_\perp sin\varphi$. The magnetic force on the wire segment is then



$$F = IlB_\perp = IlBsin\varphi$$

- The force is always perpendicular to both the conductor and the field, with the direction determined by the same ***right-hand rule***

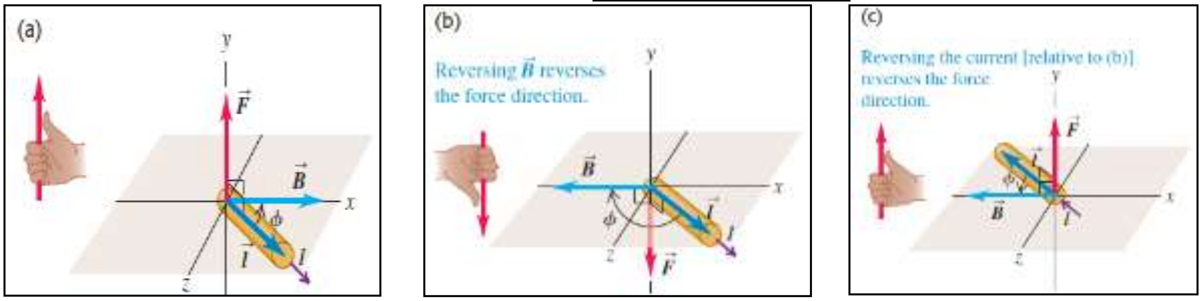
Hence this force can be expressed as a vector product, just like the force on a single moving charge. We represent the segment of wire with a vector $\boldsymbol{l}$ along the wire in the direction of the current; then the force on this segment is

$$\boldsymbol{F} = I\boldsymbol{l} \times \boldsymbol{B}$$

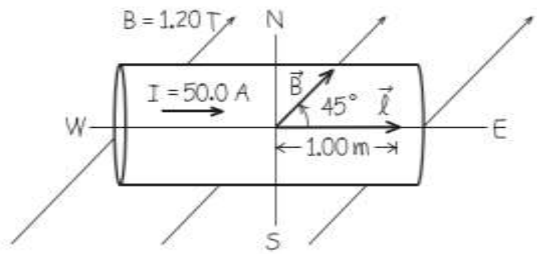
- The direction of $\boldsymbol{l}$ is the direction of the current

If the conductor is not straight, we can divide it into infinitesimal dl segments. The force dF on each segment is

$$d\boldsymbol{F} = I\, d\boldsymbol{l} \times \boldsymbol{B}$$



Example on Magnetic force on a straight conductor



A straight horizontal copper rod carries a current of 50.0 A from west to east in a region between the poles of a large electromagnet. In this region there is a horizontal magnetic field toward the northeast (that is, 45° north of east) with magnitude 1.20 T.

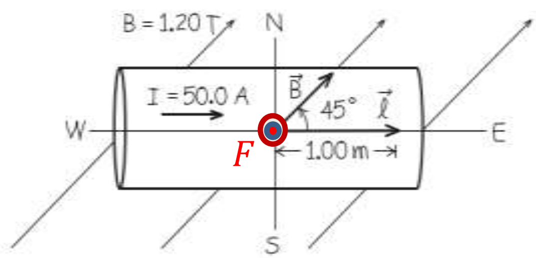
(a) Find the magnitude and direction of the force on a 1 m section of rod.

(b) While keeping the rod horizontal, how should it be oriented to maximize the magnitude of the force? What is the force magnitude in this case?

Solution: (a) The angle between the directions of current and field is 45°.

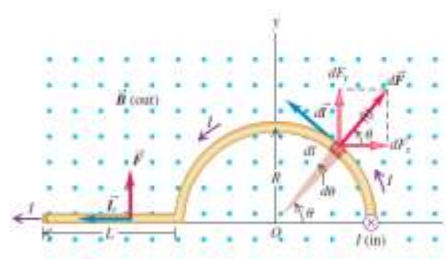
$$F = IlBsin\varphi = 50 \times 1 \times 1.2 \times sin45^0 = 42.4\ N$$

The direction of the force is perpendicular to the plane of the current and the field, both of which lie in the horizontal plane. Thus the force must be vertical; the right-hand rule shows that it is vertically upward (out of the plane of the figure).



Example on Magnetic force on a curved conductor

In the figure below the magnetic field $\boldsymbol{B}$ is uniform and perpendicular to the plane of the figure, pointing out of the page. The conductor, carrying current I to the left, has three segments: (1) a straight segment with length L perpendicular to the plane of the figure, (2) a semicircle with radius R, and (3) another straight segment with length L parallel to the x-axis. Find the total magnetic force on this conductor.



Solution: For segment (1), $\boldsymbol{L} = -L\boldsymbol{k}$. Hence $\boldsymbol{F}_1 = I\boldsymbol{L} \times \boldsymbol{B} = 0$. For segment (3), $\boldsymbol{L} = -L\boldsymbol{i}$ so $\boldsymbol{F}_3 = I\boldsymbol{L} \times \boldsymbol{B} = I(-L\boldsymbol{i}) \times (B\boldsymbol{k}) = ILB\boldsymbol{j}$. For the curved segment (2), the figure shows a segment $\boldsymbol{dl}$ with length $dl = R\,d\theta$, at angle θ. The right-hand rule shows that the direction of $\boldsymbol{dl} \times \boldsymbol{B}$, is radially outward from the center. Because $\boldsymbol{dl}$ and $\boldsymbol{B}$ are perpendicular, the magnitude $d\boldsymbol{F}_2$ of the force on the segment is just $dF_2 = I\,dl\,B = I(R\,d\theta)B$. The components of the force on this segment are

$$dF_{2x} = I\,Rd\theta\,Bcos\theta \qquad dF_{2y} = I\,Rd\theta\,Bsin\theta$$

To find the components of the total force, we integrate these expressions with respect to θ from $\theta = 0$ to $\theta = \pi$ to take in the whole semicircle. The results are

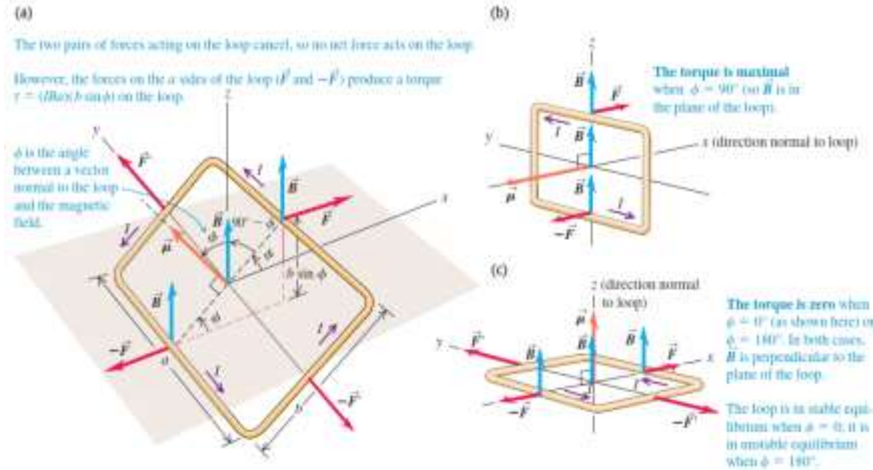
$$F_{2x} = IRB\int_0^{\pi} cos\theta\, d\theta = 0 \qquad F_{2y} = IRB\int_0^{\pi} sin\theta\, d\theta = 2IRB$$

Hence, $\boldsymbol{F}_2 = 2IRB\boldsymbol{j}$. Finally, adding the forces on all three segments, we find that the total force is in the positive y-direction:

$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 = 0 + 2IRB\boldsymbol{j} + ILB\boldsymbol{j} = IB(2R + L)\boldsymbol{j}$$

Force and Torque on a Current Loop

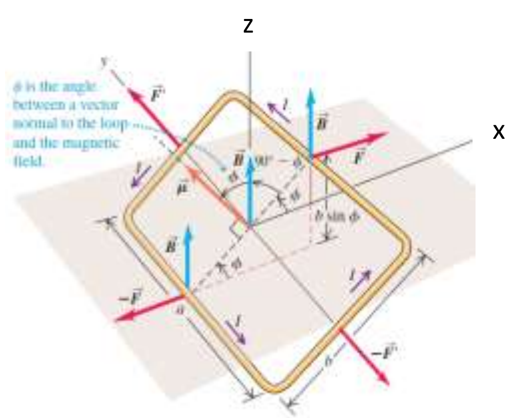
As an example, let's look at a rectangular current loop in a uniform magnetic field. We can represent the loop as a series of straight line segments. We will find that the total force on the loop is zero.



- The force on the right side of the loop (length a) is to the right, in the $+x$ –direction. B is perpendicular to the current direction,

$$F = IaB$$

- A force $-\boldsymbol{F}$ with the same magnitude but opposite direction acts on the opposite side of the loop, as shown in the figure.



- The sides with length b make an angle $(90 - \varphi)$with the direction of $\boldsymbol{B}$. The forces on these sides are the vectors $\boldsymbol{F'}$ and $-\boldsymbol{F'}$ their magnitude is given by

$$F' = IbBsin(90^0 - \varphi) = IbBcos\varphi$$

- The lines of action of both forces lie along the y-axis

$$F_{net} = F - F + F' - F' = 0$$

- The net force on a current loop in a uniform magnetic field is zero.
- However, the net torque is not in general equal to zero.

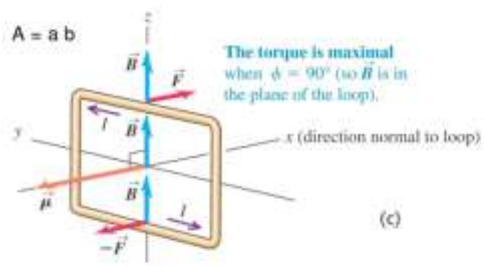
In general
$$\tau = r \times F = r_{\perp}F = rF_{\perp} = r\, F sin\varphi$$

$$\tau_F = \frac{b}{2}\, F sin\varphi = \frac{b}{2}(IaB\,) sin\varphi$$

$$\tau_{-F} = \frac{b}{2}\, F sin\varphi = \frac{b}{2}(IaB\,) sin\varphi$$

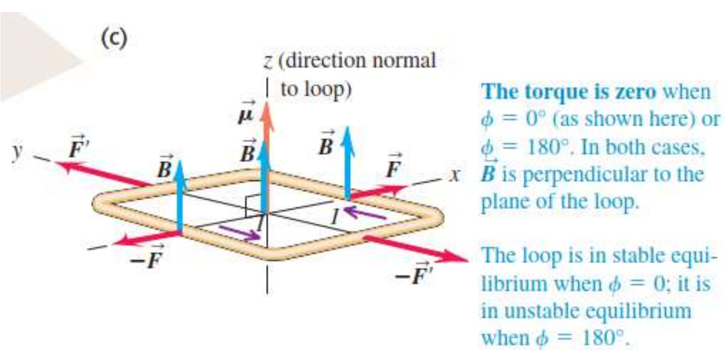
The two forces ***F'*** and ***−F'*** lie along the same line and so give rise to zero net torque with respect to any point.

$$\therefore \tau_{net} = \tau_{F'} + \tau_{-F'} + \tau_F + \tau_{-F} = 0 + 0 + 2\frac{b}{2}(IaB\,) sin\varphi = IaBbsin\varphi$$



φ is angle between a vector perpendicular to loop and ***B***

- The torque is greatest when $\varphi = 90^0$

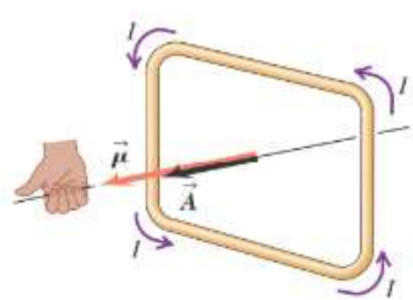


- The torque is zero when $\varphi = 0$ *or* 180^0

The area of the loop is equal to *ab* so we can rewrite the magnitude of torque on a current loop as

$$\tau = IABsin\varphi$$

- The product *IA* is called the *magnetic dipole moment* or *magnetic moment of the lo* $\mu = IA$ e use the symbol μ



- Magnetic torque $\therefore \tau_{tot} = \mu B sin\varphi$

$$\tau = \mu \times B$$

Direction: (Right Hand Rule) determines the direction of the magnetic moment of a current-carrying loop $\boldsymbol{\mu}$. This is also the direction of the loop's area vector $\boldsymbol{A}$.

- ➢ Potential Energy for a Magnetic Dipole:
- ❑ The torque on an electric dipole in an electric field is $\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E}$, we found that the corresponding potential energy is $\boldsymbol{U} = -\boldsymbol{p} \cdot \boldsymbol{E}$.
- ❑ The torque on a magnetic dipole in a magnetic field is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}$, so we can conclude immediately that the corresponding potential energy is $\boldsymbol{U} = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\mu B\cos\varphi$

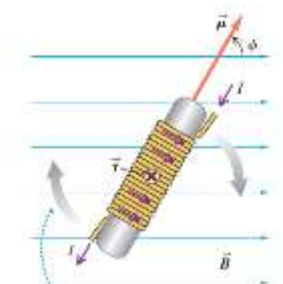
With this definition, $\boldsymbol{U}$ is zero when the magnetic dipole moment is perpendicular to the magnetic field.

Solenoid

An arrangement of particular interest is the solenoid, a helical winding of wire, such as a coil wound on a circular cylinder. If the windings are closely spaced, the solenoid can be approximated by a number of circular loops lying in planes at right angles to its long axis. The total torque on a solenoid in a magnetic field is simply the sum of the torques on the individual turns. For a solenoid with N turns with uniform field B, the magnetic moment is NIA and

$$\tau = NIABsin\varphi$$

where φ is the angle between the axis of the solenoid and the direction of the field. The magnetic moment vector μ is along the solenoid axis.



The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\mu}$ with field $\vec{B}$

Example: A circular coil 0.05 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a counterclockwise (as viewed from above) current of 5 A. The coil is in a uniform 1.2-T magnetic field directed toward the right. Find the magnitudes of the magnetic moment and the torque on the coil.

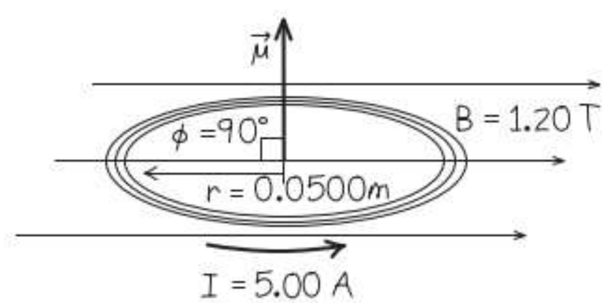
Solution: The area of the coil is $A = \pi r^2$. The total magnetic moment of all 30 turns is

$$\mu_{tot} = NIA = 30 \times 5 \times \pi(0.05)^2 = 1.18\ A.m^2$$

The angle between the direction of and the direction of (which is along the normal to the plane of the coil) is 90^0.

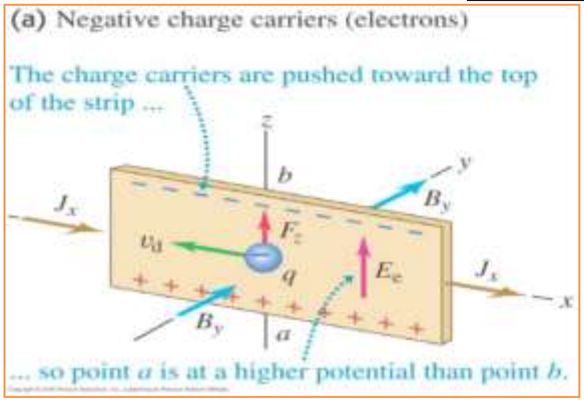
The torque on the coil is

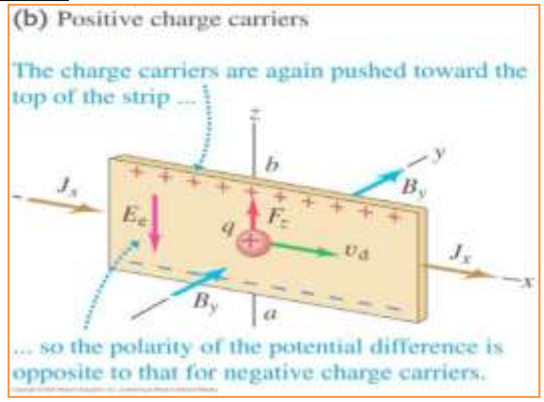
$$\tau = \mu_{tot} Bsin\,\varphi = 1.18 \times 1.2 \times \sin 90^0$$
$$= 1.41\ N.m$$



The Hall Effect

- ➢ The Hall effect: A current through a conducting material will develop a transverse voltage (Hall voltage) when the material is placed in a B-field.
- ➢ The concept is similar to the velocity selector except that the electric field ("the Hall voltage) is generated by the deflected charge carriers rather than an external E-field.





In the steady state, when the forces $F_E = qE_z$ and $F_B = qv_dB_y$ are equal in magnitude and opposite in direction,

$$qE_z + qv_dB_y = 0 \quad \text{or} \quad E_z = -v_dB_y$$

This confirms that when is positive, is negative. The current density J_x is

$$J_x = nqV_d$$

Eliminating v_d between these equations, we find

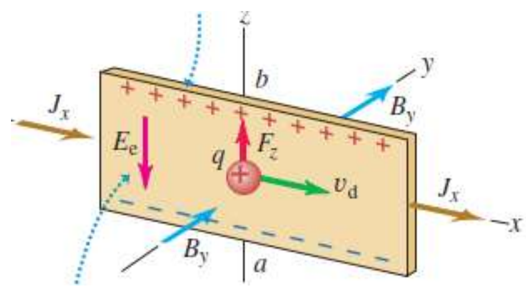
$$nq = \frac{-J_xB_y}{E_z} \quad \text{(Hall effect)}$$

Application of the Hall effect:

(1) It is easy to measure voltage; the Hall effect is used for precision measurement of magnetic field.
(2) The Hall voltage developed by positive carrier has opposite sign compared to negative carrier. The Hall Effect is used to determine the sign of the current carrier in semiconductors.

Example on A Hall-effect measurement

You place a strip of copper, 2.0 mm thick and 1.50 cm wide, in a uniform 0.40-T magnetic field as shown in the figure. When you run a 75-A current in the x-direction, you find that the potential at the bottom of the slab is 0.81 μV higher than at the top. From this measurement, determine the concentration of mobile electrons in copper.



Solution:

The current density is $J_x = \frac{I}{A} = \frac{75}{(2\times10^{-3})(1.5\times10^{-2})} = 2.5 \times 10^6\ A/m^2$

The electric field is $E_z = \frac{V}{d} = \frac{0.81\times10^{-6}}{1.5\times10^{-2}} = 5.4 \times 10^{-5}\ V/m$

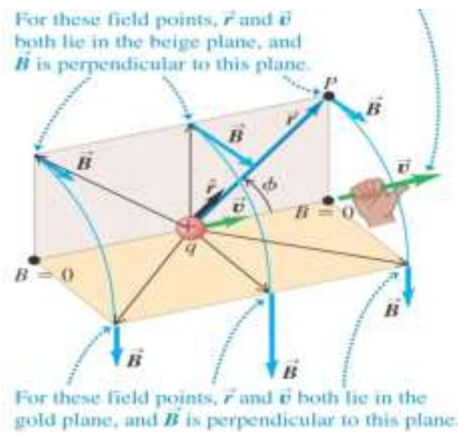
Therefore, the concentration of mobile electrons in copper is

$$n = \frac{-J_xB_y}{qE_z} = \frac{-(2.5 \times 10^6)(0.4)}{(-1.6 \times 10^{-19})(5.4 \times 10^{-5})} = 11.6 \times 10^{28} m^{-3}$$

Lecture 11. Sources of Magnetic field

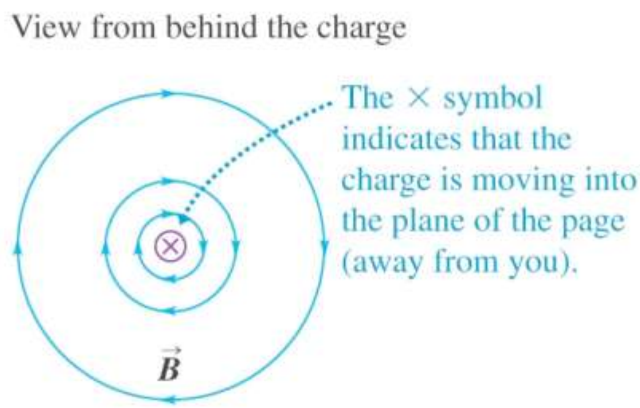
❖ *The magnetic field of a moving charge*

- A moving charge produces a magnetic field.
- The field will be perpendicular to the direction of motion of the charge.



q: source point charge
P: field point
***r** : a unit vector=1*
***v**: particle velocity vector*
***B**: magnetic field*

B is perpendicular to the plane containing the line joining *q* and *P* and the particle's velocity vector



$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\varphi}{r^2}$$

$$B = \frac{\mu_0}{4\pi}\frac{q(\boldsymbol{v}\times\boldsymbol{r})}{r^2}$$

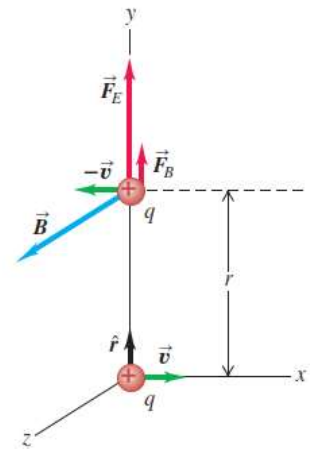
μ_0: Permeability of free space $\quad \mu_0 = 4\pi\times10^{-7}\ \frac{Tesla.meter}{Ampere}$

ε_0: Permittivity of free space $\quad \varepsilon_0 = 8.85\times10^{-12}\ \frac{Coulomb}{Newton.meter^2}$

Speed of light $\quad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8\ \ meter/second$

Example: Two protons move parallel to the x-axis in opposite directions at the same speed (small compared to the speed of light c). At the instant shown, find the electric and magnetic forces on the upper proton and compare their magnitudes.

Solution: Coulomb's law gives the electric force on the upper proton. To get the magnetic force on the upper proton, we must first find the magnetic field that the lower proton produces at the position of the upper proton. The unit vector from the lower proton (the source) to the position of the upper proton is $\boldsymbol{r} = \boldsymbol{j}$



$$F_E = \frac{1}{4\mu\varepsilon_0}\frac{q^2}{r^2}$$

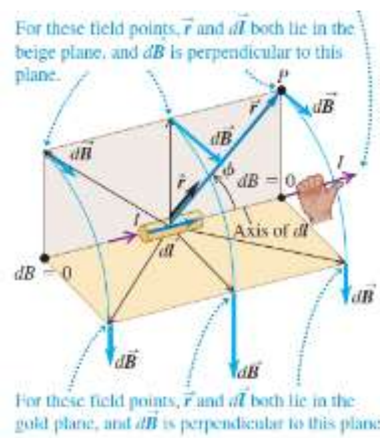
$$B = \frac{\mu_0}{4\pi}\frac{qvi \times j}{r^2} = \frac{\mu_0}{4\pi}\frac{qv}{r^2}\boldsymbol{k}$$

$$F_B = q(-\boldsymbol{v}) \times \boldsymbol{B} = -qv\boldsymbol{i} \times \frac{\mu_0}{4\pi}\frac{qv}{r^2}\boldsymbol{k} = \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}\boldsymbol{j}$$

$$\frac{F_B}{F_E} = \frac{\mu_0 q^2 v^2/4\pi r^2}{q^2/4\pi\varepsilon_0 r^2} = \frac{\mu_0 v^2}{1/\varepsilon_0} = \frac{v^2}{1/\mu_0\varepsilon_0} = \frac{v^2}{c^2}$$

The magnetic force is much smaller than the electric force because v is smaller than the speed of light.

❖ Magnetic field of a current element



the magnetic field caused by a short segment ***dl*** of a current-carrying conductor, as shown in the figure. The volume of the segment is $A\,dl$, where A is the cross-sectional area of the conductor. If there are n moving charged particles per unit volume, each of charge q, the total moving charge dQ in the segment is

$$dQ = nqAdl$$

The moving charges in this segment are equivalent to a single charge dQ, traveling with a velocity equal to the drift velocity v_d. The magnitude of the magnetic field at the field point P is

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d sin\varphi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_d A\, dl\, sin\varphi}{r^2}$$

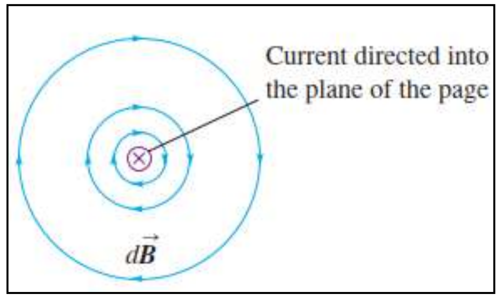
But $n|q|v_d A = I$

$$\therefore dB = \frac{\mu_0}{4\pi}\frac{I\, dl\, sin\varphi}{r^2}$$

$$\therefore dB = \frac{\mu_0}{4\pi}\frac{I\, d\boldsymbol{l} \times \boldsymbol{r}}{r^2}$$

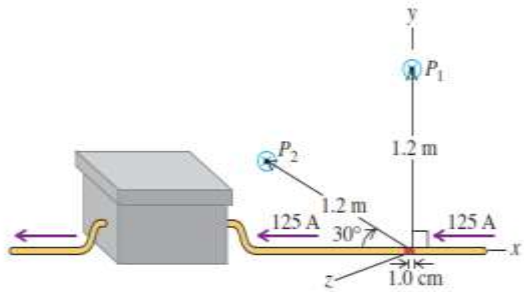
where is ***dl*** a vector with length dl, in the same direction as the current in the conductor.

- The last equations are called the ***law of Biot and Savart***
- use this law to find the total magnetic field at any point in space due to the current in a complete circuit.



Example: A copper wire carries a steady 125-A current to an electroplating tank (Figure). Find the magnetic field due to a 1.0-cm segment of this wire at a point 1.2 m away from it, if the point is

(a) Point straight out to the side of the segment, and
(b) Point in the xy-plane and on a line at to the segment.



Solution: (a) At pointP_1, the unit vector $\boldsymbol{r} = \boldsymbol{j}$

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{I\,\boldsymbol{dl}\times\boldsymbol{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl(-\boldsymbol{i})\times\boldsymbol{j}}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\boldsymbol{k}$$

$$= -(10^{-7})\frac{(124)(1\times10^{-2})}{(1.2)^2}\boldsymbol{k} = -(8.7\times10^{-8}\,T)\boldsymbol{k}$$

∴ The direction of ***B*** at P_1 is into the *xy-plane*

(b) At P_2 the unit vector is $\boldsymbol{r} = (-cos30^0)\boldsymbol{i} + (sin30^0)\boldsymbol{j}$

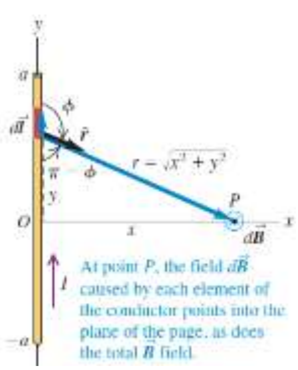
$$B = \frac{\mu_0}{4\pi}\frac{I\,\boldsymbol{dl}\times\boldsymbol{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl(-i)\times= (-cos30^0)\boldsymbol{i} + (sin30^0)\boldsymbol{j}}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dlsin30}{r^2}\boldsymbol{k} = -(10^{-7})\frac{(125)(1\times10^{-2})(sin30)}{(1.2)^2} = -(4.3\times10^{-8}\,T)\boldsymbol{k}$$

∴ The direction of ***B*** at P_2 is also into the *xy-plane*

❖ *Magnetic field of a straight current-carrying conductor*

Use the law of ***Biot and Savart*** to find the magnetic field produced by a straight current-carrying conductor.



The figure shows such a conductor with length $2a$ carrying a current I. We will find at a point a distance x from the conductor on its perpendicular bisector.

To find the field ***dB*** for the element *dl* at point P distance *x* from it.

- $dl = dy$
- $r = \sqrt{x^2 + y^2}$
- $sin\varphi = sin(\pi - \varphi) = \frac{x}{\sqrt{x^2+y^2}}$
- from RHR the direction of dB is into the plane of the figure.

The magnitude of the total magnetic field ***B*** is

$$B = \frac{\mu_0 I}{4\pi}\frac{dl\sin\varphi}{r^2} = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x\,dy}{(x^2+y^2)^{3/2}} = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}$$

$$= \frac{\mu_0 I}{2\pi}\frac{1}{x\sqrt{\frac{x^2}{a^2}+1}}$$

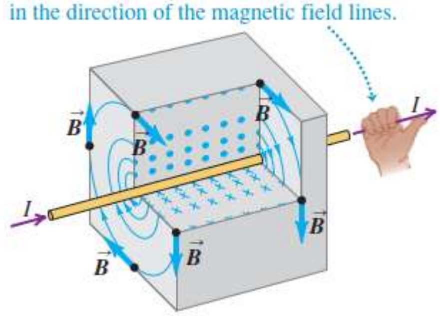
- When the length $2a$ of the conductor is very great in comparison to its distance x from the point P, we can consider it to be infinitely long.
- When a is much larger than x, $\sqrt{\frac{x^2}{a^2}+1}$ is approximately equal to 1
- Hence, in the limit $a \rightarrow \infty$

$$B = \frac{\mu_0 I}{2\pi x}$$

The physical situation has axial symmetry about the y-axis. Hence must have the same magnitude at all points on a circle centered on the conductor and lying in a plane perpendicular to it, and the direction of must be everywhere tangent to such a circle. Thus, at all points on a circle of radius r around the conductor, the magnitude B is

$$B = \frac{\mu_0 I}{2\pi r}$$

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.



Example: A long, straight conductor carries a 1.0-A current. At what distance from the axis of the conductor does the resulting magnetic field have magnitude $B = 0.5 \times 10^{-4}\ T$ (about that of the earth's magnetic field in Pittsburgh)?

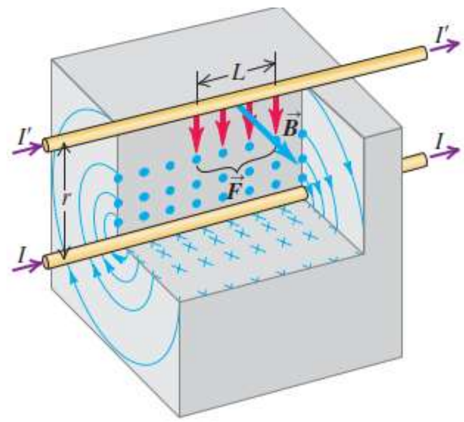
Solution:

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\therefore r = \frac{\mu_0 I}{2\pi B} = \frac{(4\pi \times 10^{-7})(1)}{2\pi(0.5 \times 10^{-4})}$$
$$= 4 \times 10^{-3} m = 4\ mm$$

❖ Force between parallel conductors

The figure shows segments of two long, straight, parallel conductors separated by a distance r and carrying currents I and I' in the same direction. Each conductor lies in the magnetic field set up by the other, so each experiences a force. The figure shows some of the field lines set up by the current in the lower conductor.



The lower conductor produces a B field that, at the position of the upper conductor, has magnitude

$$B = \frac{\mu_0 I}{2\pi r}$$

he force that this field exerts on a length L of the upper conductor is $\boldsymbol{F} = I'\boldsymbol{L} \times \boldsymbol{B}$ where the vector $\boldsymbol{L}$ is in the direction of the current I' and has magnitude L. Since $\boldsymbol{B}$ is perpendicular to the length of the conductor and hence to $\boldsymbol{L}$ the magnitude of this force is

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r}$$

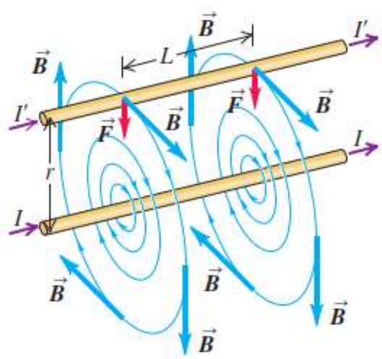
and the force per unit length F/L is

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} \quad \text{(two long, parallel, current-carrying conductors)}$$

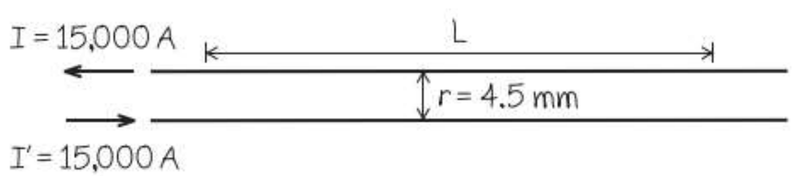
Applying the right-hand rule to $\boldsymbol{F} = I'\boldsymbol{L} \times \boldsymbol{B}$ shows that the force on the upper conductor is directed downward.

The current in the upper conductor also sets up a field at the position of the lower one.

Thus *two parallel conductors carrying current in the same direction attract each other. If the direction of either current is reversed, the forces also reverse. Parallel conductors carrying currents in opposite directions repel each other.*



Example: Two straight, parallel, superconducting wires $4.5\ mm$ apart carry equal currents of $15{,}000\ A$ in opposite directions. What force, per unit length, does each wire exert on the other?



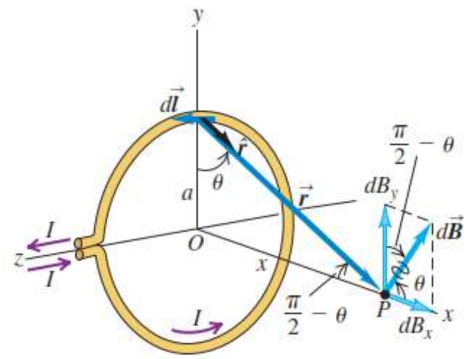
Solution: The conductors repel each other because the currents are in opposite directions.
The force per unit length is

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} = \frac{(4\pi \times 10^{-7})(15 \times 10^{3})^{2}}{2\pi(4.5 \times 10^{-3})}$$
$$= 1 \times 10^{4}\ N/m$$

❖ *Magnetic field of a circular current loop*

Use the law of Biot and Savart to find the magnetic field at a point P on the axis of the loop, at a distance x from the center. As the figure shows, $\boldsymbol{dl}$ and $\boldsymbol{r}$ are perpendicular, and the direction of the field $d\boldsymbol{B}$ caused by this particular element lies in the xy-plane. Since $r^2 = x^2 + a^2$ the magnitude dB of the field due to element $\boldsymbol{dl}$ is

$$d\boldsymbol{B} = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2 + a^2)}$$



The components of the vector dB are

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2 + a^2)}\frac{a}{(x^2 + a^2)^{1/2}}$$
$$dB_x = dB\sin\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2 + a^2)}\frac{x}{(x^2 + a^2)^{1/2}}$$

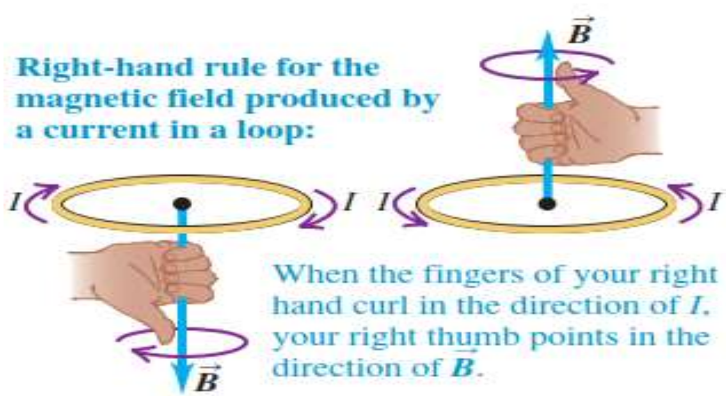
The total field $\boldsymbol{B}$ at P has only an x-component (it is perpendicular to the plane of the loop).
To obtain the x-component of the total field we integrate around the loop. Everything in this expression except dl is constant and can be taken outside the integral, and we have

$$B_x = \int \frac{\mu_0 I}{4\pi}\frac{a\,dl}{(x^2 + a^2)^{3/2}} = \frac{\mu_0 I}{4\pi}\frac{a}{(x^2 + a^2)^{3/2}}\int dl$$

The integral of dl is just the circumference of the circle, $\int dl = 2\pi a$ and we finally get

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \qquad \text{(on the axis of a circular loop)}$$

The direction of the magnetic field on the axis of a current-carrying loop is given by a right-hand rule. If you curl the fingers of your right hand around the loop in the direction of the current, your right thumb points in the direction of the field.



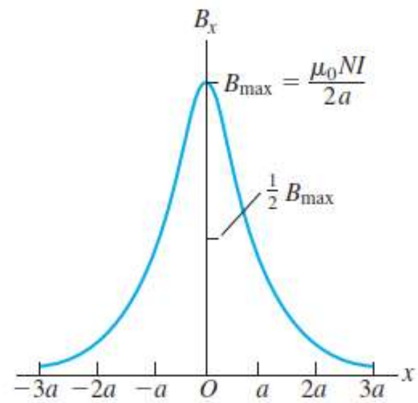
❖ *Magnetic Field on the Axis of a Coil*

Now suppose that instead of the single loop, we have a coil consisting of N loops, all with the same radius. Then the total field is N times the field of a single loop:

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}} \quad \text{(on the axis of } N \text{ circular loops)}$$

The maximum value of the field is at the center of the loop or coil at $x = 0$

$$B_{max} = \frac{\mu_0 N I}{2a}$$



Example: A coil consisting of 100 circular loops with radius 0.60 m carries a 5.0-A current.

(a) Find the magnetic field at a point along the axis of the coil, 0.80 m from the center.

(b) Along the axis, at what distance from the center of the coil is the field magnitude $\frac{1}{8}$ as great as it is at the center?

Solution:

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$$

(a) At $x = 0.8\ m$ from the center

$$B_x = \frac{(4\pi \times 10^{-7})(100)(5)(0.6)^2}{2(0.8^2 + 0.6^2)^{3/2}} = 1.1 \times 10^{-4}\ T$$

(b) we want to find a value of x such that

$$\frac{1}{(x^2 + a^2)^{3/2}} = \frac{1}{8}\frac{1}{(0^2 + a^2)^{3/2}}$$

$$(x^2 + a^2)^{2/3} = 8(a^2)^{2/3}$$

$$x^2 + a^2 = 4a^2$$

$$\therefore x = \pm\sqrt{3}\ \ a = 1.04\ \text{m}$$

❖ *Ampere's law*

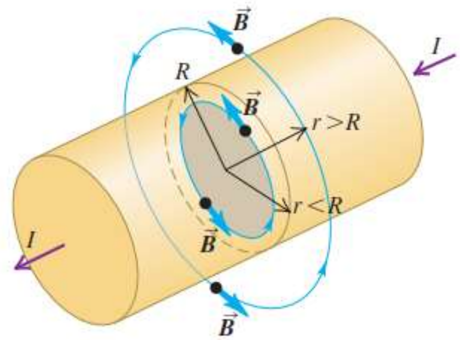
- Ampère's Circuital Law relates the magnetic field to its electric current source.
- Ampere's law allows us to calculate magnetic fields from the relation between the electric currents that generate this magnetic fields. It states that *for a closed path the sum over elements of the component of the magnetic field is equal to electric current multiplied by the permeability of free space.*
- It is the law that a magnetic field induced by an electric current is, at any point, *directly proportional to the product of the current and the length of the current conductor, inversely proportional to the square of the distance between the point and the conductor, and perpendicular to the plane joining the point and the conductor.*
- Ampere's law is formulated not in terms of magnetic flux, but rather in terms of the line integral of around a closed path, denoted by

$$\oint B \cdot dl = \mu_0 I$$

Example: A cylindrical conductor with radius R carries a current I. The current is uniformly distributed over the cross-sectional area of the conductor. Find the magnetic field as a function of the distance r from the conductor axis for points both inside and outside the conductor.

Solution: In either case the field ***B*** has the same magnitude at every point on the circular integration path and is tangent to the path. Thus the magnitude of the line integral is simply $B(2\pi r)$. To find the current I_{encl} enclosed by a circular integration path inside the conductor $(r < R)$, note that the current density (current per unit area) is,

$$J = I/\pi R^2$$



so the $I_{encl} = J(\pi r^2) = Ir^2/R^2$

Hence Ampere's law gives

$$B(2\pi r) = \mu_0 I r^2/R^2$$

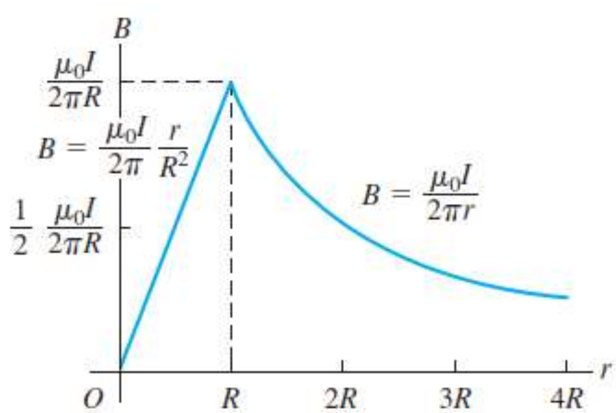
- ❑ The field inside the conductor $r < R$

$$\therefore B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}$$

Outside the conductor the integration encloses the total current in the conductor, so $I_{encl} = I$

- ❑ The field outside the conductor $r > R$

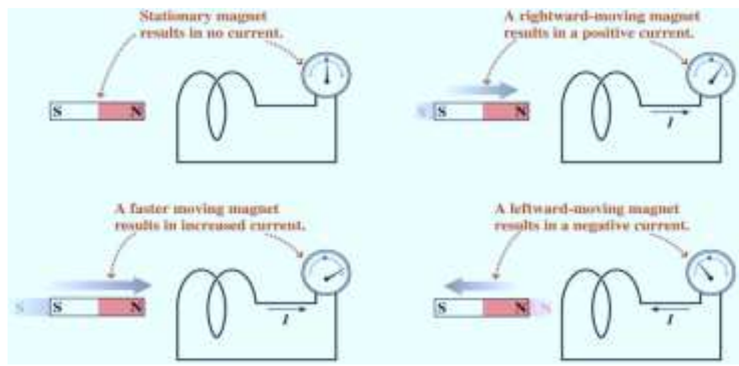
$$\therefore B = \frac{\mu_0 I}{2\pi r}$$



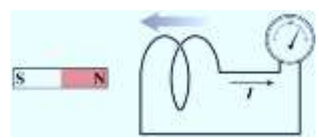
Lecture 12. Electromagnetic induction

- **Electromagnetic induction is the process of using magnetic fields to produce voltage, and in a complete circuit, a current.**
- The current in the coil induced by a changing magnetic field or changing the area of a coil methods is called an ***induced current***. A closed circuit is necessary for the induced current to flow.
- The emf produced in the coil which drives the induced current is called the "induced emf". The induced *emf* exists whether or not the coil is part of a closed circuit.
- The phenomenon of producing an induced emf with the aid of a magnetic field is called ***electromagnetic induction.***

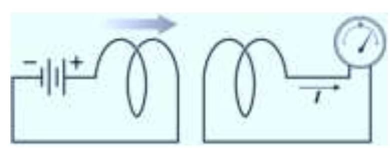
  - Simple experiments show that it doesn't matter how the magnetic field changes: Induced electrical effects occur in all cases of changing magnetic fields.



- **Experiment 2: moving circuit/coil near a magnet; an induced current results**

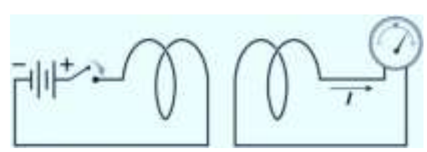


- **Experiments 3 and 4: two circuits; either one moving**
  - (3) Energize one coil to make it an electromagnet; move it near a circuit and induced current results.
  - (4) Energize one coil to make it an electromagnet; hold it stationary and move a circuit near it—an induced current results.



- **Experiment 5: changing field/current; no motion**

Change the current in one circuit, and thus the magnetic field it produces; induced current results in a nearby circuit



❖ *Magnetic Flux*

To understand the complex nature of electromagnetic induction is to understand the idea of magnetic flux.

Flux is a general term associated with a FIELD that is bound by a certain AREA. So MAGNETIC FLUX is any AREA that has a MAGNETIC FIELD passing through it.

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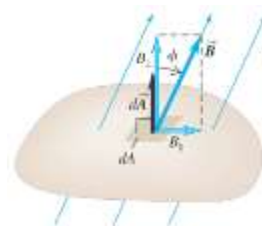
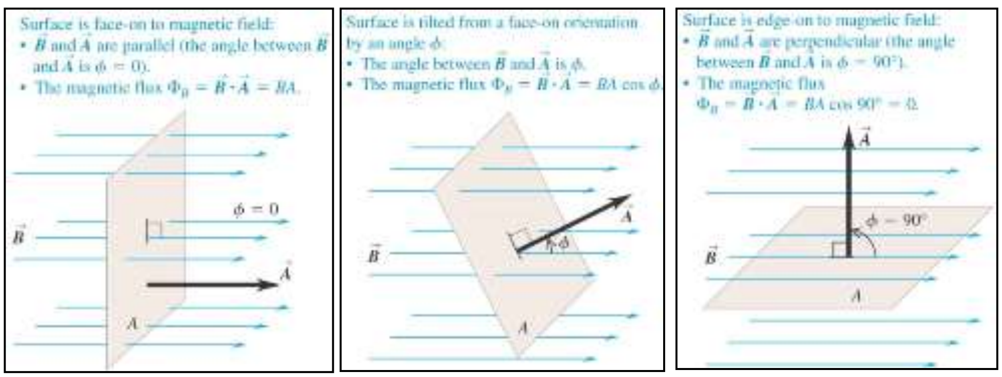
We generally define an AREA vector as one that is perpendicular to the surface of the material. Therefore, you can see in the figure that the AREA vector and the Magnetic Field vector are PARALLEL. This then produces a DOT PRODUCT between the 2 variables that then define flux.

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos\varphi \cdot dA$$



If B is uniform over a flat area A:

$$\Phi_B = \vec{B} \cdot \vec{A} = B \cdot A \cdot \cos\varphi$$



❖ ***Faraday's Law of Induction:***

The induced ***emf*** in a closed loop equals the negative of the time rate of change of the magnetic flux through the loop.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

Increasing flux ⟹ $\varepsilon < 0$

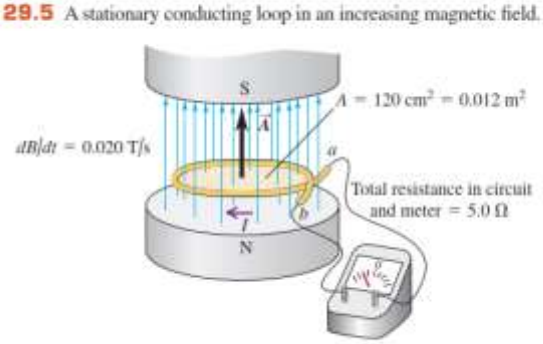
Decreasing flux ⟹ $\varepsilon > 0$

Example1: The magnetic field between the poles of the electromagnet in Fig. 29.5 is uniform at any time, but its magnitude is increasing at the rate of The area of the conducting loop in the field is 120 cm2, and the total circuit resistance, including the 0.020 T/s. meter, is

(a) Find the induced emf and the induced current in the circuit.
(b) If the loop is replaced by one made of an insulator, what effect does this have on the induced emf and induced current?

Solution:

29.5 A stationary conducting loop in an increasing magnetic field.



(a)The area vector ***A*** for the loop is perpendicular to the plane of the loop; we take ***A*** to be vertically upward. Then ***A*** and ***B*** are parallel, and because ***B*** is uniform the magnetic flux through the loop is

$\boldsymbol{\phi_B} = \boldsymbol{B} \cdot \boldsymbol{A} = BA\cos 0 = BA$. The area $\boldsymbol{A} = 0.012\ m^2$ is constant, so the rate of change of magnetic flux is .

$$\frac{d\boldsymbol{\phi_B}}{dt} = \frac{d(BA)}{dt} = \frac{dB}{dt}A = 0.02 \times 0.012 = 2.4 \times 10^{-4} V$$

This, apart from a sign that we haven't discussed yet, is the induced ***emf*** ε . The corresponding induced current is

$$I = \frac{\varepsilon}{R} = \frac{2.4 \times 10^{-4}}{5} = 4.8 \times 10^{-5} A$$

(b) By changing to an insulating loop, we've made the resistance of the loop very high. Faraday's law does not involve the resistance of the circuit in any way, so the induced ***emf*** does not change. But the current will be smaller. If the loop is made of a perfect insulator with infinite resistance, the induced current is zero. This situation is analogous to an isolated battery whose terminals aren't connected to anything:

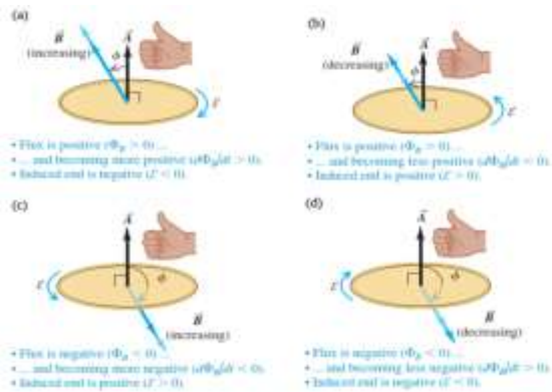
An emf is present, but no current flows.

❖ *Direction of Induced emf:*

We can find the direction of an induced emf or current by using the Faraday's law of induction $\varepsilon = -\frac{d\phi_B}{dt}$, together with some simple sign rules. Here's the procedure:

1. Define a positive direction for the vector area ***A***.

2. From the directions of ***A*** and the magnetic field ***B*** determine the sign of the magnetic flux $\boldsymbol{\phi_B}$ and its rate of change $d\boldsymbol{\phi_B}/dt$



3. Determine the sign of the induced ***emf*** or current. If the flux is increasing, so $d\boldsymbol{\phi_B}/dt$ is positive, then the induced ***emf*** or current is negative; if the flux is decreasing, $d\boldsymbol{\phi_B}/dt$ is negative and the induced ***emf*** or current is positive.

4. Finally, determine the direction of the induced ***emf*** or current using your right hand. Curl the fingers of your right hand around the ***A*** vector, with your right thumb in the direction of ***A***. If the induced ***emf*** or current in the circuit is *positive*, it is in the same direction as your curled fingers; if the induced ***emf*** or current is *negative*, it is in the opposite direction.

For a coil with *N* identical turns, and if the flux varies at the same rate through each turn, the *total rate* of change through all the turns is N times that for a single turn. If $\boldsymbol{\phi_B}$ is the flux through each turn, the total emf in a coil with N turns is $\varepsilon = -N\frac{d\phi_B}{dt}$

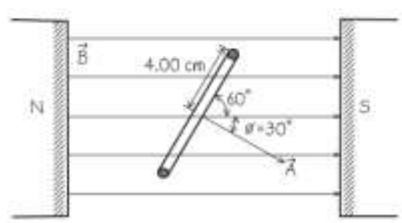
Example 2: A 500-loop circular wire coil with radius *4 cm* is placed between the poles of a large electromagnet. The magnetic field is uniform and makes an angle of 60^0 with the plane of the coil; it decreases at $0.2\ T/s$. What are the magnitude and direction of the induced *emf*?

Solution: The flux varies because the magnetic field decreases in amplitude. We choose the area vector ***A*** to be in the direction shown in the figure below. With this choice, the geometry is similar to (b) of the direction figure above. Since the magnetic field is uniform, then the magnetic flux is

$$\boldsymbol{\phi_B} = BA\cos\varphi$$

Where $\varphi = 30^0$.

(Remember that φ is the angle between ***A*** and ***B*** not the angle between ***B*** and the plane of the loop.)



Therefore, the induced emf in the coil

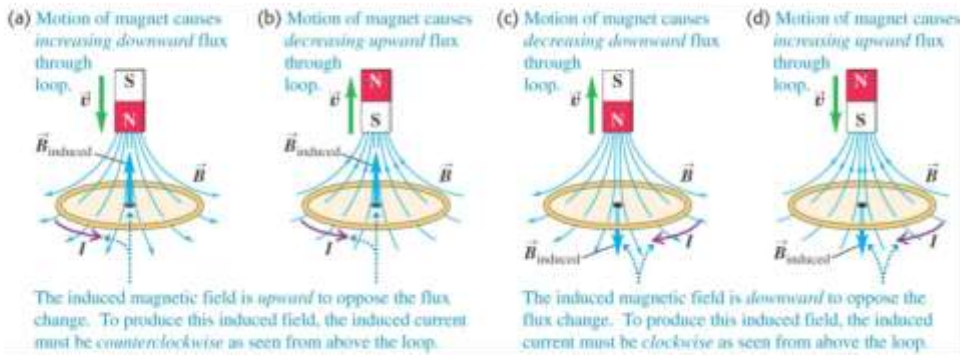
$$\varepsilon = -N\frac{d\phi_B}{dt} = -N\frac{dB}{dt}A\cos\varphi$$
$$= 500(-0.2)(\pi\ 0.04^2)(\cos 30) = 0.435\ V$$

The positive answer means that when you point your right thumb in the direction of the area vector ***A*** (below the magnetic field), the positive direction for ε is in the direction of the curled fingers of your right hand.

❖ Lenz's Law

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

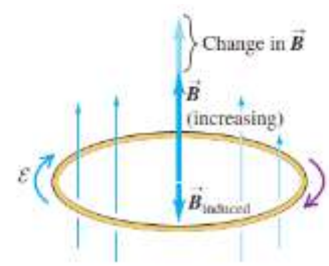
- Alternative method for determining the direction of induced current or emf.
- The "cause" can be changing the flux through a stationary circuit due to varying B, changing flux due to motion of conductors, or both.



If the flux in an stationary circuit changes, the induced current sets up a magnetic field opposite to the original field *if original **B** increases*, but in the same direction as original B *if **B** decreases*.

- *The induced current opposes the change in the flux through a circuit* (not the flux itself).
- If the change in flux is due to the motion of a conductor, the direction of the induced current in the moving conductor is such that the direction of the magnetic force on the conductor is opposite in direction to its motion (e.g. slide-wire generator). *The induced current tries to preserve the "status quo" by opposing motion or a change of flux.*

B induced downward opposing the change in flux ($d\Phi/dt$). This leads to induced current clockwise.

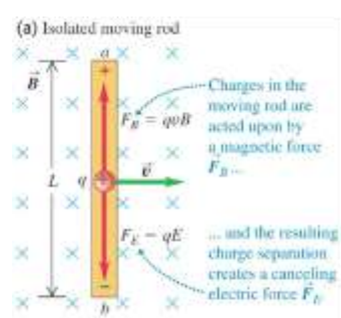


❖ *Lenz's Law and the Response to Flux Changes*

- Lenz's Law gives only the direction of an induced current I. The magnitude depends on the circuit's resistance. Large R → small induced I → easier to change flux through circuit.
- If loop is a good conductor → I induced present as long as magnet moves with respect to loop. When relative motion stops → I = 0 quickly (due to circuit's resistance).
- If R = 0 (superconductor) → I induced (persistent current) flows even after induced emf has disappeared (after magnet stopped moving relative to loop). The flux through loop is the same as before the magnet started to move → flux through loop of $R = 0$ does not change.

❖ *Motional Electromotive Force*

A charged particle in rod experiences a magnetic force $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$ that causes free charges in rod to move, creating excess charges at opposite ends.



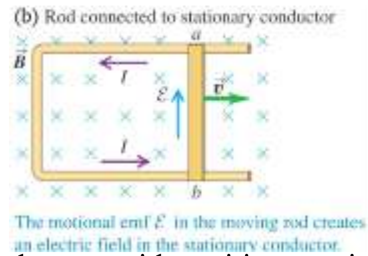
- The excess charges generate an electric field (from a to b) and electric force ($F = q\,E$) opposite to magnetic force.
- Charge continues accumulating until F_E compensates F_B and charges are in equilibrium $q\,E = q\,v\,B$
- the magnitude of the potential difference $V_{ab} = V_a - V_b$ is equal to the

electric field magnitude E multiplied by the length L of the rod.

$$V_{ab} = EL = vBL$$

- If rod slides along stationary U-shaped conductor forming a complete circuit. No magnetic force acts on charges in U-shaped conductor, but excess charge at ends of straight rod redistributes along U-conductor, creating an electric field.

The electric field in stationary U-shaped conductor creates a current moving rod became a source of *emf* (*motional electromotive force*). Within straight rod charges move from lower to higher potential, and in the rest of circuit from higher to lower potential.



$$\varepsilon = vBL$$

Length of rod and velocity perpendicular to ***B***.

Induced current:

$$I = \frac{\varepsilon}{R} = \frac{vBL}{R}$$

- The emf associated with the moving rod is equivalent to that of a battery with positive terminal at a and negative at b.
- Motional *emf*: general form (alternative expression of Faraday's law)

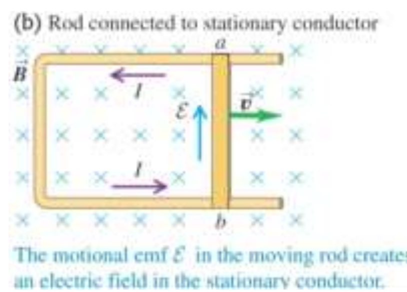
$$d\varepsilon = (v \times B) \cdot dl$$

$$\varepsilon = \oint (v \times B) \cdot dl \qquad \text{Closed conducting loop}$$

- This expression can only be used for problems involving moving conductors. When we have stationary conductors in changing magnetic fields, we need to use: $\varepsilon = -\,d\phi_B/dt$

Example: Suppose the moving rod in the figure below is $0.10\ m$ long, the velocity is $v = 2.5\ m/s$, the total resistance of the loop is $R = 0.03\ \Omega$ and B is $0.60\ T$. Find the motional emf, the induced current, and the force acting on the rod.

Solution: the motional emf is



$$\varepsilon = vBL = 2.5 \times 0.6 \times 0.1 = 0.15\ V$$

The induced current in the loop is

$$I = \frac{\varepsilon}{R} = \frac{0.15}{0.03} = 5\ A$$

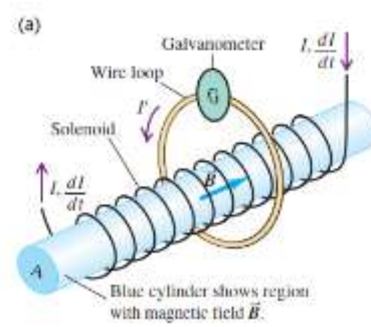
the magnetic force acting on the rod has magnitude

$$F = ILB = 5 \times 0.1 \times 0.6 = 0.3\ N$$

Since ***L*** and ***B*** are perpendicular, the magnetic force $\boldsymbol{F} = I\boldsymbol{L} \times \boldsymbol{B}$, by the RHR is directed opposite to the rod 's motion.

❖ *Induced Electric Fields*

- An induced emf occurs when there is a changing magnetic flux through a stationary conductor.
- A current (I) in solenoid sets up ***B*** along its axis, the magnetic flux is:



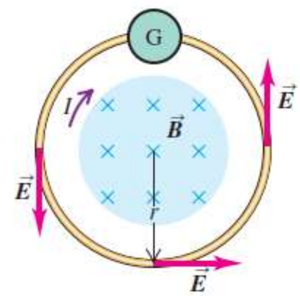
$$\Phi_B = B \cdot A = \mu_0 n I A$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt}$$

Induced current in loop (I'):

$$I' = \frac{\varepsilon}{R}$$

- The force that makes the charges move around the loop is not a magnetic force. There is an *induced electric field* in the conductor caused by a changing magnetic flux.
- The total work done on q by the induced E when it goes once around the loop: $= q\varepsilon$, therefore E is not conservative.
- For conservative E: $\oint \boldsymbol{E} \cdot d\boldsymbol{l} = \mathbf{0}$
- For non-conservative E: $\oint \boldsymbol{E} \cdot d\boldsymbol{l} = \boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt}$ (stationary path)
- Cylindrical symmetry E magnitude constant, direction is tangent to loop.



$$\oint \vec{E} \cdot d\vec{l} = 2\pi \cdot r \cdot E \longrightarrow E = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right|$$

Example: Suppose a long solenoid has 500 turns per meter and cross-sectional area $4cm^2$. The current in its windings is increasing at 100 A/s

(a) Find the magnitude of the induced emf in the wire loop outside the solenoid.

(b) Find the magnitude of the induced electric field within the loop if its radius is 2.0 cm.

Solution: (a) the induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\mu_0 nA\frac{dI}{dt}$$

$= -(4\pi \times 10^{-7})(500)(4 \times 10^{-4})(100)$
$= -25 \times 10^{-6} Wb/s$
$= -25 \times 10^{-6} V = -25\mu V$

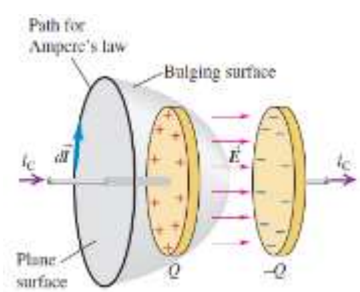
(b) By symmetry the line integral $\oint \boldsymbol{E} \cdot d\boldsymbol{l}$ has absolute value $2\pi rE$ no matter which direction we integrate around the loop. This is equal to the absolute value of the emf, so

$$|\varepsilon| = 2\pi rE$$

$$E = \frac{|\varepsilon|}{2\pi r} = \frac{25 \times 10^{-6}}{2\pi(2 \times 10^{-2})} = 2 \times 10^{-4} V/m$$

❖ *Displacement Current and Maxwell's Equations*

- A varying electric field gives rise to a magnetic field. the magnetic field can be obtained by using Ampere's law



$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I_{encl}$$

Where I_{encl} is the conduction current passing through surface by closed path.

- Consider charging a capacitor: Conducting wires carry i_c (conduction current) into one plate and out of the other, as Q and E between plates increase. for the circular path shown apply Ampere's law to find

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 i_c$$

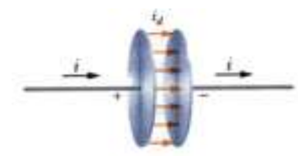
Consider a second surface that bulges out to the right which is also bounded by the same circle, the current through that surface is zero, because the charge stops on the capacitor plates. So $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 i_c$ and at the same time it is equal to zero! *This is a clear contradiction.*

- ❑ As capacitor charges, $\boldsymbol{E}$ and Φ_E through surface increase.

- The instantaneous charge on the plates is $q = Cv$, where v is the instantaneous potential difference across the plates.

$$C = \varepsilon_0 \frac{A}{d} \quad \text{and} \quad v = Ed \quad \text{so}$$

$$q = Cv = \varepsilon_0 \frac{A}{d}(Ed) = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

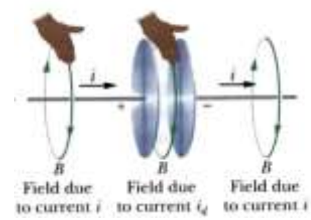


$$i_c = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

we invent a fictitious ***displacement current*** in the region between the plates, defined as

$$i_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

To generalize Ampere's law, we include this fictitious current, along with the real conduction current



$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 (i_c + i_D)_{encl}$$

Ampere's law in this form is obeyed no matter which surface we use. For the flat surface, i_D is zero; for the curved surface, i_C is zero; and

$$i_C(flat\ surface) = i_D(curved\ surface)$$

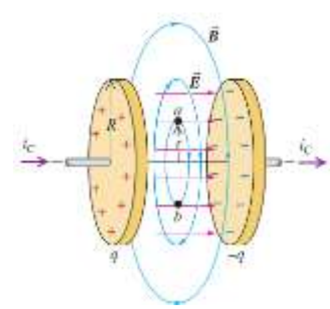
- **Displacement current density (j_D):**

$$j_D = \frac{i_D}{A} = \varepsilon \frac{dE}{dt}$$

- The displacement current is the source of B in between capacitor's plates. It helps us to satisfy Kirchhoff's junction's rule: i_C in and i_D out

❖ ***The reality of Displacement Current***

Displacement current creates ***B*** between plates of capacitor while it charges.
Let's picture round capacitor plates with radius *R*. To find the magnetic field at a point in the region between the plates at a distance *r* from the axis, we apply Ampere's law to a circle of radius *r* passing through the point, with $r < R$
This circle passes through points *a* and *b*.
The total current enclosed by the circle is j_D times its area, or $(i_D/\pi R^2)(\pi r^2)$. The integral in Ampere's law is just B times the circumference $2\pi r$ of the circle, and because for the charging capacitor, Ampere's law becomes



$$\oint B \cdot dl = 2\pi r B = \mu_0 \frac{r^2}{R^2} i_C$$

$$B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_C$$

This result predicts that in the region between the plates ***B*** *is* zero at the axis and increases linearly with distance from the axis. A similar calculation shows that outside the region between the plates (that is, for $r > R$) ***B*** is the same as though the wire were continuous and the plates not present at all.

❖ *Maxwell's Equations of Electromagnetism*

➤ Gauss's law for electricity $\oint E \cdot dA = \frac{Q_{encl}}{\varepsilon_0}$ it describes charges and electric field.

It physically means that

- like charges repel and unlike charges attract,
- A charge on an insulated conductor moves to its outer surface.

➤ Gauss's law for magnetism $\oint B \cdot dA = 0$ it describes the magnetic field.

It physically means that

- There are no magnetic monopoles

➤ Ampere's law (as extended by Maxwell) $\oint B \cdot dl = \mu_0 \left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt}\right)_{encl}$ it describes the magnetic effect of a current or a changing electric field.

It physically means that

- A current in a wire sets up a magnetic field near the wire.
- The speed of light can be calculated from purely electromagnetic measurements.

➤ Faraday's law of induction $\oint E \cdot dl = -\frac{d\Phi_E}{dt}$ it describes the electrical effect of changing magnetic field.

It physically means that

- A bar magnet thrust through a closed loop of wire will set up a current in the loop.